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MULTIPURPOSE SINGLE RESERVOIR OPERATION UNDER FUZZY ENVIRONMENT WITH FUZZY RESOURCES AND FUZZY TECHNOLOGICAL COEFFICIENTS

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Abstract: *Relatively little of the research on reservoir operating procedures has found its way in actual practice. One reason is that operators are uncomfortable with complex optimization models and reluctant to use procedures that they do not fully understand. Also in classical optimization model, the objective function and the constraints are represented by very precisely under uncertainty. However, many of the constraints are externally controlled and the variations cannot be predicted to a reliable extent. This may cause difficulties in representing these interacting variables for optimizations. Fuzzy logic approach seems to offer a way to improve on existing operating practices, which is relatively easy to explain and understand. This paper presents an optimal reservoir operation using fuzzy logic approach to a multi-objective linear programming problem. A fuzzy linear programming based model is developed for optimal operation of a multipurpose single reservoir. Model considers fuzzy nature involved in the objectives as well as in resources and technological coefficients of the constraints. Model considers two objective functions viz. maximization of total releases for irrigation and maximization of total releases for hydropower production. The steps involved in the development of model include the fuzzification of the objective functions, construction of membership function for the demands, releases, storage and objective functions and maximization of level of satisfaction of objective functions. The methodology is illustrated through the case study of the Jayakwadi reservoir Stage-II, built on river Sindaphana, a tributary of river Godavari, Maharashtra State, India. Reservoir storage, inflows and demands are used as premises and the releases as the consequence. Optimal operating policy has been developed for maximized level of satisfaction (* λ *) in a water year.*

Keywords: *Reservoir operation, Fuzzy linear programming, Multi-objective analysis, irrigation releases, hydropower production.*

INTRODUCTION

Impact of climate changes are likely to give rise to uncertainties in water availability and water demands, which may result in major economics and economical consequences. Uncertainty can be crucial for water availability for domestic, agricultural, or industrial uses, and therefore is, an important parameter in developing reservoir system models. Unfortunately in the real world most of the parameters used are very uncertain. Therefore even if the linear programming formulation is accepted, neither the constraints, nor the revenues, can be characterized by

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certainty (Tsakiris and Spiliotis, 2004). Panigrahi and Mujumdar (2000) developed a reservoir operation model with fuzzy logic. In this study, a fuzzy rule based model is developed for the operation of single purpose reservoir. Labadie (2004) has given the state-of-the-art review for optimal operation of multireservoir systems. The purpose of this review is to access the state-of-the-art in optimization of reservoir system management and operations and consider future directions for additional research and application. Srinivasa Raju and Nagesh Kumar (2000) demonstrated that how vagueness and imprecision in the objective function values can be quantified by membership function in a fuzzy multiobjective framework. Recent research in modelling uncertainty in water resource systems has highlighted the use of fuzzy logic-based approaches (Mujumdar and Ghosh, 2008). Nagesh Kumar *et al.* (2001) have developed the optimal reservoir model using multi objective fuzzy linear programming (MOFLP). A linear membership functions are used to fuzzify the objective functions. Only objectives are taken to be fuzzy and all other parameters of the model are considered crisp in nature. Regulwar and Anand Raj (2008) developed 3-D optimal surface for operation policies of a multireservoir in a fuzzy environment using Genetic Algorithm (GA) for river basin development and management. These objectives are fuzzified and are simultaneously maximized by defining and then maximizing level of satisfaction (λ). Amiri *et al.* (2009) have considered two classes of fuzzy linear programming (FLP) problems: Fuzzy number linear programming and linear programming with trapezoidal fuzzy variables problems. Study gives recently established results and develop fuzzy primal simplex algorithms for solving these problems. Nasseri (2008) has given a new method for solving FLP by solving linear programming. He has used some vector computations on fuzzy vectors, where a fuzzy vector appears as a vector of triangular fuzzy numbers. Wenguang and Yunling (2009) proposed a method for solving FLP problems where all the coefficients are fuzzy numbers. The most popular approach to handle the challenge of solving fuzzy linear programming problems is to convert the FLP into the corresponding deterministic linear programming. Study uses the complementary slackness to solve it without the need of a simplex tableau (Ebrahimnejad and Nasseri, 2009). Gasimov and Yenilmes (2002) have solved FLP problems with linear membership function. Ganesan and Verramani (2006) introduced a new type of fuzzy arithmetic for symmetric trapezoidal fuzzy numbers and propose a method for solving FLP problems without converting them to crisp linear programming problems. In many practical situations, it is not reasonable to require that the constraints or the objective function in linear programming problems be specified in precise, crisp terms. In such situations, it is desirable to use some type of FLP (Klir and Yuan, 2000). Regulwar and Anand Raj (2009) have studied multiobjective multireservoir optimization in fuzzy environment for river basin development and management. A model is proposed using GA under fuzzy environment. The optimal operation policy obtained by the model is compared with the actual average operation policy for Jayakwadi reservoir stage – I. Loucks et al. (1981) has given development and application of quantitative mathematical modelling methods to problems of water management. Form the literature review it can be said that the fuzzy logic can be used to incorporate the uncertainty, imprecise or qualitative decision making problems. Most of the literature considered either fuzzy objectives or fuzzy resources in FLP model. Objective of the present study is to decide the sequences of optimal releases for the multipurpose single reservoir.

In the present study a FLP model is developed by considering fuzzy technological coefficients and the fuzzy resources along with fuzzy objectives. A linear membership function is used to fuzzify the objectives as well as constraints. The model is solved for maximizing the satisfaction level (λ) of the objectives considered in the study. The FLP model is demonstrated through case study of Jayakwadi Reservoir Stage – II built across river Sindaphana, Maharashtra State, India.

METHODOLOGY

Fuzzy Linear Programming Problem with fuzzy technological coefficients and fuzzy resources

In the following problem FLP model with fuzzy technological coefficients and fuzzy resources have presented.

$$\max = \sum_{j=1}^{n} c_j x_j$$

$$\sum_{j=1}^{n} \tilde{a}_{ij} x_j \le \tilde{b}_i \qquad 1 \le i \le m \tag{1}$$

$$x_j \ge 0,$$

$\tilde{a}_{ij}$ and $\tilde{b}_i$ are fuzzy numbers with the following linear membership function:

$$\mu_{a_{ij}}(x) = \begin{cases} 1 & if \ x < a_{ij}, \\ (a_{ij} + d_{ij} - x)/d_{ij} & if \ a_{ij} \le x \le a_{ij} + d_{ij}, \\ 0 & if \ x \ge a_{ij} + d_{ij}, \end{cases} \tag{2}$$

and

$$\mu_{b_i}(x) = \begin{cases} 1 & if \ x < b_i, \\ (b_i + p_i - x)/p_i & if \ b_i \le x \le b_i + p_i, \\ 0 & if \ x \ge b_i + p_{i,} \end{cases} \tag{3}$$

Where $x \in \mathbb{R}$. For defuzzification of the problem given by Eq. (1), we first calculate the lower and upper bounds of the optimal values. The optimal values z_l and z_u can be defined by solving the following standard linear programming problems, for which we assume that all they have the finite optimal values.

$$z_1 = \max \sum_{j=1}^{n} c_j x_j$$

$$\sum_{j=1}^{n} (a_{ij} + d_{ij}) x_j \le b_i, \qquad 1 \le i \le m \tag{4}$$

$$x_j \ge 0,$$

$$z_2 = \max \sum_{j=1}^{n} c_j x_j$$

$$\sum_{j=1}^{n} a_{ij} x_j \le b_i + p_i, \qquad 1 \le i \le m \tag{5}$$

$$x_j \ge 0,$$

$$z_3 = \max \sum_{j=1}^{n} c_j x_j$$
$$\sum_{j=1}^{n} \left(a_{ij} + d_{ij}\right) x_j \le b_i + p_i, \qquad 1 \le i \le m \tag{6}$$
$$x_j \ge 0,$$

and

$$z_4 = \max \sum_{j=1}^{n} c_j x_j$$
$$\sum_{j=1}^{n} a_{ij} x_j \le b_i, \qquad 1 \le i \le m \tag{7}$$
$$x_j \ge 0,$$

Let $z_l = \min(z_1, z_2, z_3, z_4)$ and $z_u = \max(z_1, z_2, z_3, z_4)$. The objective function takes values between z_l and z_u while technological coefficients takes values between a_{ij} and $a_{ij} + d_{ij}$ and the right side numbers (resources) takes values between b_i and $b_i + p_i$. Then the fuzzy set of optimal values, G, which is fuzzy subset of $\mathbb{R}^n$ is defined by a linear membership function $G(x)$ for objective as,

$$\mu_G(x) = \begin{cases} 0 & \text{if } \sum_{j=1}^{n} c_j x_j < z_l, \\ \left(\sum_{j=1}^{n} c_j x_j - z_l\right) \Big/ (z_u - z_l) & \text{if } z_l \le \sum_{j=1}^{n} c_j x_j < z_u, \\ 1 & \text{if } \sum_{j=1}^{n} c_j x_j \ge z_u, \end{cases} \tag{8}$$

Graphical representation of linear membership function given by Eq. (8) for fuzzy goals is shown in Fig. 1.

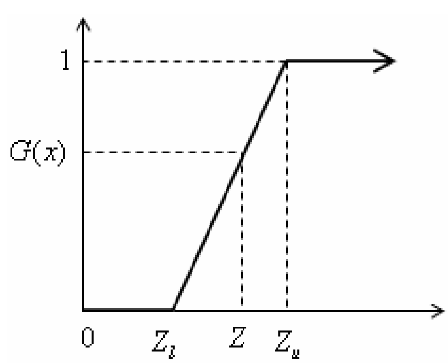

Fig. 1. Linear membership function for goals.

The fuzzy set of i^{th} constraint, C_i which is subset of $\mathbb{R}^n$ is defined by

$$\mu c_i(x) = \begin{cases} 0 & \text{if } b_i < \sum_{j=1}^{n} a_{ij}x_j, \\ \left(b_i - \sum_{j=1}^{n} a_{ij}x_j\right) \Big/ \left(\sum_{j=1}^{n} d_{ij}x_j + p_i\right) & \text{if } \sum_{j=1}^{n} a_{ij}x_j \le b_i < \sum_{j=1}^{n}(a_{ij}+d_{ij})x_j + p_i, \\ 1 & \text{if } b_i \ge \sum_{j=1}^{n}(a_{ij}+d_{ij})x_j + p_i, \end{cases} \tag{9}$$

The linear membership function used for fuzzy technological coefficients ($\tilde{a}_{ij}$) and fuzzy resources ($\tilde{b}_i$) given by Eq. (9) is represented graphically in Fig. 2.

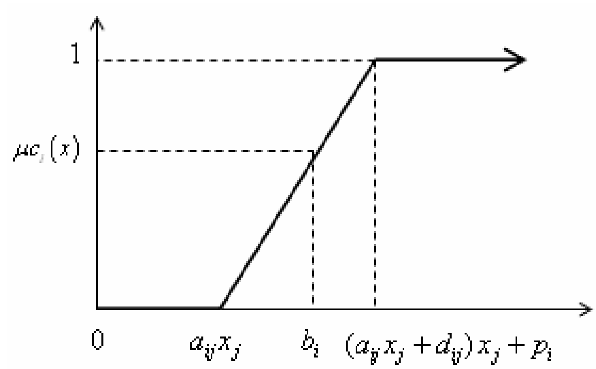

Fig. 2. Linear membership function for both fuzzy technological coefficients ($\tilde{a}_{ij}$) and fuzzy resources ($\tilde{b}_i$).

By incorporating the above information, the problem given by Eq. (1) becomes the following optimization problem

$$\begin{aligned} &\max \lambda \\ &s.t.\ \mu G(x) \ge \lambda, \\ &\quad \mu C_i(x) \ge \lambda, \qquad 1 \le i \le m \\ &\quad x \ge 0, 0 \le \lambda \le 1. \end{aligned} \tag{10}$$

By using Eq. (8) and (9), Eq. (10) can be written as

$$\begin{aligned} &\max \lambda \\ &\lambda(z_u - z_l) - \sum_{j=1}^{n} c_j x_j + z_l \le 0 \\ &\sum_{j=1}^{n}(a_{ij} + \lambda d_{ij})x_j + \lambda p_i - b_i \le 0, \qquad 1 \le i \le m \\ &\qquad x \ge 0, 0 \le \lambda \le 1. \end{aligned} \tag{11}$$

Where, λ is the satisfaction level.

A reservoir operation model is developed by using above FLP formulation and applied to the case study of Jayakwadi reservoir Stage-II. Model is solved for maximization of satisfaction level (λ) of both the objectives and results have been obtained to decide an optimal operating policy in a water year.

CASE STUDY

The physical system considered is the Jayakwadi reservoir stage-II, a multipurpose project, created by constructing a dam across the river Sindaphana, a tributary of river Godavari, in Aurangabad district, Maharashtra State, India as shown in Fig. 3. Inflows into the Jayakwadi reservoir stage-II consist of feeder canal releases from Jayakwadi reservoir stage-I and runoff from upstream catchments. The gross storage of reservoir is 453.64 Mm^3 and live storage is 313.30 Mm^3. The total installed capacity for power generation is 2.25MW. Irrigable command area is 938.85 km^2.

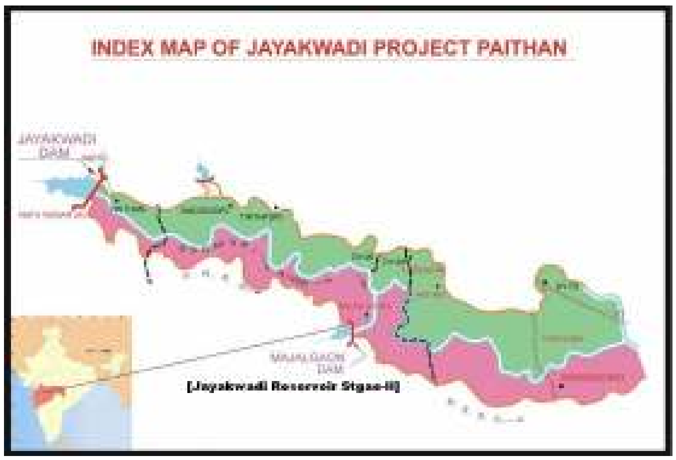

Fig. 3. Location of Jayakwadi reservoir stage – II.

The 75% dependable monthly inflows into the reservoir are shown in Table 1. Monthly irrigation demands were determined with the help of crop calendar, water requirements for various crops during different growth stages and the types of soils. Monthly irrigation demands in a water year are also shown in Table 1.

Table 1. Monthly inflows and irrigation demands for Jayakwadi reservoir stage – II

Month	Inflows in Mm^3	Irrigation demand in Mm^3
Jun	20.98	7.12
Jul	43.46	20.83
Aug	96.88	37.64
Sep	144.17	46.02
Oct	75.52	132.01
Nov	10.24	127.05
Dec	4.27	89.43
Jan	0.37	100.68
Feb	0.37	30.02
Mar	0.16	28.98
Apr	0.12	35.58
May	0.06	25.88

FORMULATION OF FLP MODEL

Following generalized LP model is developed for monthly operation of the reservoir assuming stationary inflows in a water year. As explained in methodology, FLP formulations are incorporated in following generalized linear programming model.

Objective Function

The two objectives are considered in the model are:

(1) Maximization of releases for irrigation (RI) and (2) Maximization of releases for hydropower production (RP).

$Max\ Z_1 = Max(TOTRI)$ and $Max\ Z_2 = Max(TOTRP)$

Where ***TOTRI*** is the total releases for irrigation in all the time periods and ***TOTRP*** is the total releases for hydropower production. These objective functions can be written as,

$$Max\ Z_1 = \sum_{t=1}^{12} RI_t$$
$$Max\ Z_2 = \sum_{t=1}^{12} RP_t \qquad (12)$$

Constraints

Turbine release constraint

Release for the turbine for hydropower production should be less than or equal to turbine capacity (TC) in each month (t), and it should be greater than or equal to the releases required to produce firm power (FP) committed for that month.

$$\begin{aligned} &RP_t \le TC && \forall t = 1,2,.....,12 \\ &RP_t \ge FP_t && \forall t = 1,2,.....,12 \end{aligned} \qquad (13)$$

Irrigation demand constraint

Release into canals for irrigation (RI) should be less than or equal to maximum irrigation demand (ID). Release should also be greater than minimum releases required for irrigation so that crop will not wilt. In the present case 30% of the irrigation demand is considered as minimum irrigation demand for all the time periods.

$$\begin{aligned} &RI_t \le ID_t && \forall t = 1,2,.....,12 \\ &RI_t \ge ID_{MIN\,t} && \forall t = 1,2,.....,12 \end{aligned} \qquad (14)$$

Reservoir storage capacity constraint

The live storage in the reservoir should be less than or equal to the maximum capacity (SC) and greater than or equal to minimum storage capacity (S_{Min}) for all the time periods.

$$\begin{aligned} &S_t \le SC && \forall t = 1,2,.....,12 \\ &S_t \ge S_{Min} && \forall t = 1,2,.....,12 \end{aligned} \qquad (15)$$

Reservoir storage continuity constraint

These constraint relate to the releases for the turbine $(RP)_t$, releases for irrigation $(RI)_t$, reservoir storage $(S)_t$, inflow$(I)_t$ into the reservoir, feeder canal release $(FCR)_t$, overflows $(OVF)_t$ and the evaporation losses $(L)_t$ for all the time periods. Here evaporation losses are considered as a function of storage and by assuming a linear relationship between reservoir water surface area and storage, storage continuity constraint can be written as follows.

$$(1-a_t)\, S_t + I_t + FCR_t - RI_t - RP_t - A_0 e_t - OVF_t = (1+a_t) S_{t+1} \qquad (16)$$

Where,

$a_t = A_a e_t / 2$

A_a is surface area of the reservoir per unit active storage volume.

A_o is surface area of the reservoir corresponding to the dead storage volume.

e_t is evaporation rate for month t in depths units.

The linear programming model is formulated in this section is applied to the case study, and is solved using LINGO (Language for Interactive General Optimization). Results are obtained by considering both fuzzy technological coefficients and fuzzy resources in the model.

RESULTS AND DISCUSSIONS

In this study the FLP model is developed and is applied to the Jayakwadi reservoir stage – II. The two objectives are considered in the model viz. maximization of releases for irrigation (RI) and maximization of releases for hydropower production (RP). As demonstrated in methodology, the model considers uncertainty involved in resources ($\tilde{b}_i$) i.e. the turbine capacity (TC), maximum irrigation demands (ID_{max}) and maximum storage capacity (SC) of the reservoir and in technological coefficients ($\tilde{a}_{ij}$) i.e. releases for irrigation (RI), releases for hydropower production (RP) and storage in the reservoir (S) in any time period t are considered fuzzy in nature. By adopting the methodology explained, using Eq. (4) to Eq. (7) the lower bound Z_l and upper bound Z_u for both the objectives (Viz, Z_1: Releases for Irrigation and Z_2: Releases for Hydropower) are determined by considering one objective at a time. These values are given in Table 2. When Z_1 is maximized, the corresponding value of Z_2 considered being worst and vice versa.

Table 2. Upper and Lower bounds of the objective function

Bounds	Objective Functions	
	Releases for Irrigation (Mm^3)	Releases for Hydropower (Mm^3)
Maximum (Upper Bound)	475.00	348.00
Minimum (Lower Bound)	204.3720	104.40

Once the upper and lower bounds of the objective functions are determined, next, objective functions are fuzzified by using linear membership function given by Eq.(8). The linear membership function for both the objectives Z_1 and Z_2 are shown in Fig. 4 and Fig. 5 respectively and can be defined as follows.

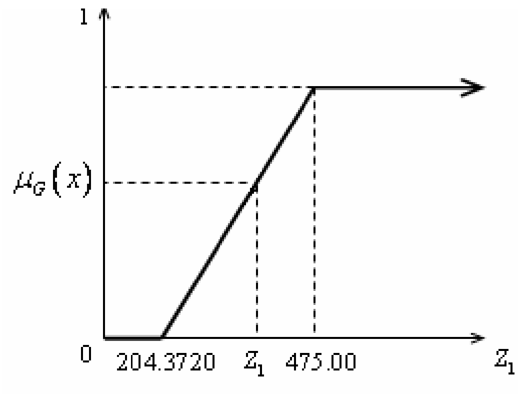

Fig. 4. Membership Function for Z_1.

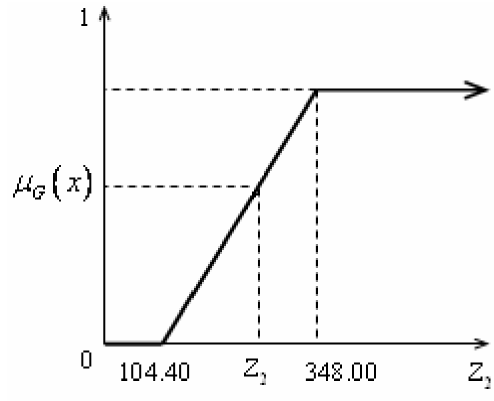

Fig. 5. Membership Function Z_2.

$$\mu_{Z_1}(x)=\begin{cases}0 & if\ Z_1 \le 204.3720 \\ (Z_1-204.3720/(475.00-204.3720) & if\ 204.3720 \le Z_1 \le 475.00 \\ 1 & if\ Z_1 \ge 475.00\end{cases} \tag{18}$$

$$\mu_{Z_2}(x)=\begin{cases}0 & if\ Z_2 \le 104.40 \\ (Z_2-104.40/(348.00-104.40) & if\ 104.40 \le Z_2 \le 348.00 \\ 1 & if\ Z_2 \ge 348.00\end{cases} \tag{19}$$

Similarly Eq. (9) is used to fuzzify the constraints. By using above information, Eq. (11) is used to solve the model for the maximization of the satisfaction level (λ) for both the objectives. Results so obtained are given in Table 3. Maximum satisfaction level (λ) obtained by solving Eq. (11) is equal to 0.525 and corresponding annual releases for irrigation are 346.47 Mm^3 and that for hydropower are 232.31 Mm^3.

Table 3. Release policy for both fuzzy resources ($\tilde{b}_i$), fuzzy technological coefficients ($\tilde{a}_{ij}$)

Months	Fuzzy resources ($\tilde{b}_i$) and fuzzy technological coefficients ($\tilde{a}_{ij}$)	
	Release for irrigation (Mm^3)	Release for hydropower (Mm^3)
June	2.136	9.4937
July	10.03273	24.17679
August	29.9563	24.17679
September	36.62564	24.17679
October	105.0619	24.17679
November	69.4946	24.17679
December	26.829	24.17679
January	30.204	24.17679
February	9.006	24.17679
March	8.694	12.005
April	10.67	8.7
May	7.764	8.7
Total	**346.47417**	**232.31302**

From the results obtained it can be seen that the in the month of June, February, March, April and May the releases for irrigation are 2.316 Mm^3, 9.006 Mm3, 8.694 Mm^3, 10.67 Mm^3 and 7.764 Mm^3 respectively. For these particular months the FLP model has satisfied minimum irrigation requirements. Also in the month of August, September and October releases obtained for irrigation are 80% of that maximum irrigation demand. Also in the month of June, April and May, FLP model has satisfied firm power requirement however releases for hydropower production in all the remaining months are more than 80% of that turbine capacity. From the obtained results it can be said that the releases are uniformly distributed over the time period for both the objectives. Release policy obtained for both objectives i.e. for irrigation and hydropower have been shown graphically in Fig. 6.

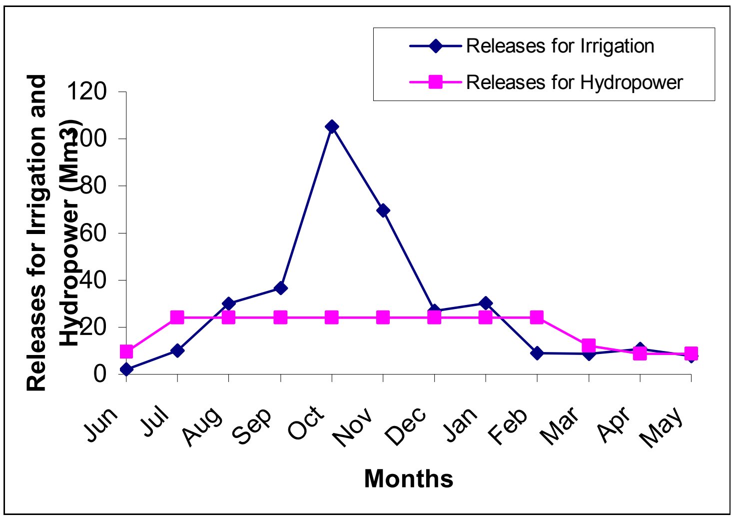

Fig. 6. Releases for irrigation and hydropower.

CONCLUSIONS

Most of the literature review considered the fuzziness, uncertainty or vagueness involved in either objective and/or in resources. In this study, a FLP model is developed for reservoir operation where model considers both technological coefficients and resources are characterized by uncertainty. A linear membership function is used to fuzzify objectives as well as constraints. Satisfaction level (λ) achieved by the model is equal to 0.53. In real world situations, it is not feasible to presume that uncertainty is involved either only in technological constraints or resources. However present FLP model has considered uncertainty comprehensively in all the parameters of reservoir operation model by considering fuzzy technological coefficients and fuzzy resources which is more near to the real world situations.

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REFERENCES

Amiri, N.M., Nasseri, S.H., and Yazdani, A. 2009. Fuzzy primal simplex algorithms for solving fuzzy linear programming problems. *Iranian J. of Operation Research,* 1(2), 68-84.

Ganesan, K. and Veeramani, P. 2006. Fuzzy linear programming with trapezoidal fuzzy numbers. *J. Ann. Operation Research,* 143, 305-315.

Gasimov, R. N. and Yenilmenz, K. 2002. Solving fuzzy linear programming problem with linear membership function. *Turk Journal of Mathematics,* 26, 375-396.

Klir, G J. and Yuan, B. 2000 *Fuzzy sets and fuzzy logic*. Prentice Hall, India.

Labadie, J.W.2004. Optimal operation of multireservoir system: State of the art review. *J. Water Resour. Plan. and Mange.,* 130, 93-111.

Loucks, D. P., Stedinger, J. and Haith, D. A. 1981. *Water resources systems planning and analysis.* Prentice-Hall, Eaglewood Cliffs, NJ.

Mujumdar, P.P. and Ghosh, S. 2008. Fuzzy logic based approaches in water resources system modeling. *Wat.Sci. Technol.* 68, 165-176.

Nageshkumar, D., Prasad, S. V. and Srinivasa R. K. 2001. Optimal reservoir operation using fuzzy approach. *Proceedings Of International Conference on Civil Engineering, Banglore, 373-384.*

Nasseri, S.H. 2008. A new method for solving fuzzy linear programming by solving linear programming. *Applied Mathematical Sciences,* 2(50), 2473-2480.

Panigrahi, D.P. & Mujumdar, P.P. 2000. Reservoir operation modeling with fuzzy logic. *Water Resour. Manag*. 14, 89-109.

Regulwar, D. G. and Anand Raj, P. 2008. Development of 3-D optimal surface for operation policies of a multireservoir in fuzzy environment using genetic algorithm for river basin development and management. *Water Resour. Manag*. 22, 595-610.

Regulwar, D. G. and Anand Raj, P. 2009. Multi objective multireservoir optimization in fuzzy environment for river basin development and management. *J. Water Resource and Protection*, 4, 271-280.

Srinivasa Raju, K. and Nagesh Kumar, D. 2000. Irrigation planning of Sri Ram Sagar project using multiobjective fuzzy linear programming. *J. Hydraulic Engg.,* 6(1), 55-62.

Tsakiris, G. and Spiliotis, M. 2004. Fuzzy linear programming problem of water allocation under uncertainty. *European Water,* 7/8, 25-27.

Wenuang, T. and Yunling, L. 2009. A new method of fuzzy linear programming. *International Conference on Business Intelligence and Financial Engineering,* 179-181.