Numerical modelling of graded sediment transport based on the experiments of Wilcock and Crowe (2003)

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Abstract—**The current work focuses on the sediment transport of non cohesive graded sediment with the TELEMAC-MASCARET Modelling System. The 2D hydro-sedimentary numerical models are based on the experiments lead by Wilcock and Crowe [2003] ran in straight flumes where both water and sediment are recirculated, using a wide range of flows and grain sizes distributions (GSD). Motivations of this study lie on (*i*) the improvement of the transport rates estimation in the code with comparison to the classical formulation for bedload transport of MPM [1948] and (*ii*) the applicability of a graded sediment transport model to the numerical simulation of complex morphodynamic problems such as bar formation and propagation. Results show that the computed sediment fluxes are strongly sensitivite to the method of discretization of the GSD, and satisfactory transport rates can be obtained with a relevant discretization of the GSD.**

I. Introduction

Modelling the transport of mixtures of *non-cohesive* particles (also referred to as *graded sediment*) remains a challenge due to the difficulty to reproduce the non-linear interactions between grains of different shape and size. In the last decades, a variety of sediment transport models have been proposed [7], but most of them are based on experimental data and no general physics-based formula for sediment transport capacity exists, although some works linking phenomenon of turbulence to sediment mobilization are promising [9].

Two types of model have been proposed to estimate the sediment transport capacity of graded sediment : stochastic models and deterministic models. Stochastic models for mixed sediment are based on the computation of fractional mobility of sediment by using the concept of continuous-time Markov process (for more information, the reader is referred to [21], [26]). Stochastic models remained quite limited, as the model parameters (*e.g.* particles velocity) have to be determined from exhaustive field campaigns or experiments. Moreover, stochastics model have to be improved, such as taking into account the spatial and temporal variation of bed shear stress and sand content [26].

Deterministic models were first proposed to estimate the sediment transport capacity of *unisize* (also known as *uniform*) sediment [6], [7], [10]. Natural sand-gravel-bedded rivers often show a wide Grain Size Distribution (GSD), and application of classical bedload equations usually fails in reproducing relevant transport rates [17]. This failure occurs because of the strong interactions between fine and coarse grains, which play a major role on the sediment transport process [4], [5], [13]. This process is commonly refered to as the *hiding* or *hiding-exposure* effect, which is manifested by the nature of coarse grains to hide finer grains in their interstices. While the gravitational effects make the larger particles harder to move, the hiding effects tends to counterbalance this phenomenon by increasing coarse grain mobility and decreasing the mobility of fine grains. Simplest models proposed the calculation of a *hiding-exposure* coefficient for each size fraction of sediment (initially proposed in the seminal work of Einstein [5]), to be replaced in the classical sediment transport capacity formula for uniform sediment to estimate a Shields parameter and a *fractional transport rate* for each size fraction [14]. Today, those formulations are commonly used in numerical modelling of graded sediment processes [20].

However, neither of those classical transport formulations were based on data derived from beds of heterogeneous sediment, and there is no justification for assuming that either equation can simply be applied with the Shields parameter based on d_i (the grain size of the i^{th} class) multiplied by F_i (the volume fraction of the i^{th} class of sediment). The use of those classical approaches would be far more justifiable by using a representative grain size d (the median grain size or the geometric mean grain size). If a fractional transport procedure is to be used, it should be done with one of the formulations derived for that purpose [18] (and references therein). These more sophisticated models take account of the hiding effects of poorly-sorted sediment by calculating a fractional-based bed load transport based on a similarity analysis between the bed load transport rate of size fraction i denoted $q_{b,i}$ and with the skin friction relative to the same size fraction denoted τ_{ri} [3], [12], [16], [23]. The similarity hypothesis assumes that transport rates are determined by the same transport law for each fraction [3], [7].

In the framework of modelling alluvial bar formation, development and stabilization, it is of main interest to fairly estimate the sediment transport in natural rivers, since rivers commonly show a variability on the GSD [1] and bars evolution (*i.e.* armor formation and *break-up*) depend on fractional transport rates estimation [11], [12], [15]. In other words, the simulation of bar dynamics often raises issues that can have an impact on the estimation of bar's characteristics such as their length, height, or temporality. Therefore, the authors proposed to implement and reproduce numerically the experiments of Wilcock and Crowe [23] (WC-2003) using a two dimensional fully-nonlinear physics based numerical model using the Telemac-Mascaret modelling system (TMS).

The model of Wilcock and Crowe [23] is interesting in the way that *i*) it is based on surface investigations and is particularly adapted for the prediction of transient conditions of bed armoring and scenarios of bed aggradation/degradation, *ii*) it considers the full size distribution of the bed surface (from finest sands to coarsest gravels), *iii*) it was calibrated using a total of 49 flume experiments with small-to-high water discharges and five different sediment mixtures and later modified and validated with 6239 values of Q_s, and *iv*) the hiding function has been designed to resolve discrepancies observed from previous experiments [12], [16] including the hiding exposure effect of sand content on gravel transport for weak to high values of sand content in the bulk mix.

Although the sophisticated formulas would be powerful tools as sediment transport estimators, it is yet not clear how to use it in the scope of numerical modelling. For example, previous numerical studies using the WC-2003 formula are based on the discretization of the GSD into only 2 size classes of bed material corresponding to sand and to gravel respectively [2], [22], whereas natural rivers usually show a continuous spectrum of grain sizes. Therefore, the numerical application of such models raises several problems in matter of GSD discretization: *i*) which method should be used to discretize the GSD? and *ii*) does the number of size classes of sediment plays an important role in the estimation of transport rates? The present paper aims to investigate these two points.

A description of the mathematical models (hydrodynamics and morphodynamics) and of the numerical treatment of physical processes is provided in Part II, together with the experimental data from WC-2003 used in this numerical study. In Part III, attention is given on the numerical modelling of graded sediment transport in the goal of reproducing the laboratory experiments of WC-2003. Results provided by the numerical models are analyzed and compared with experimental data in Part IV.

II. Materials and Methods

A. Two-dimensional hydrodynamic model

The hydrodynamics solver Telemac-2D is internally coupled to the sediment transport and bed evolution module Sisyphe. The hydrodynamics module is based on the solution of shallow-water equations (SWE) obtained from several strong assumptions (hydrostatic pressure distribution, averaged vertical velocity, *etc.*), wherein the momentum diffusion coefficient is assumed equal to the turbulent viscosity, which is constant troughout the domain with $\nu_t = 10^{-6}$ m^2/s:

$$\begin{cases} \partial_t h + \vec{u}\cdot\vec{\nabla}(h) + h div(\vec{u}) = S_h \\ \partial_t u + \vec{u}\cdot\vec{\nabla}(u) = -g\partial_x z_s - S_{f,x} + h^{-1} div(h\nu_t \vec{\nabla} u) \\ \partial_t v + \vec{u}\cdot\vec{\nabla}(v) = -g\partial_y z_s - S_{f,y} + h^{-1} div(h\nu_t \vec{\nabla} v) \end{cases} \tag{1}$$

where $\partial_t = \partial/\partial_t$, g is the acceleration of gravity = 9.81 m.s^{-2}, h is the water depth [m], $z_s = z_b + h$ the free surface [m] (Fig.1) with z_b the elevation of the riverbed topography [m], $\vec{u} = (u, v)$ with $|\vec{u}|$ the module of $\vec{u}$, u (resp. v) the fluid velocity along the Cartesian x-axis (resp. y-axis) [m/s] and $S_{f,x}$ (resp. $S_{f,y}$) corresponds to the friction forces along the Cartesian x-axis (resp. y-axis). In this work, the friction coefficient is determined with the law of Strickler:

$$S_{f,i} = \frac{u_i|\vec{u}|}{K^2 h^{4/3}} \quad , \tag{2}$$

where K is the friction coefficient of Strickler and i is stands as the Einstein index.

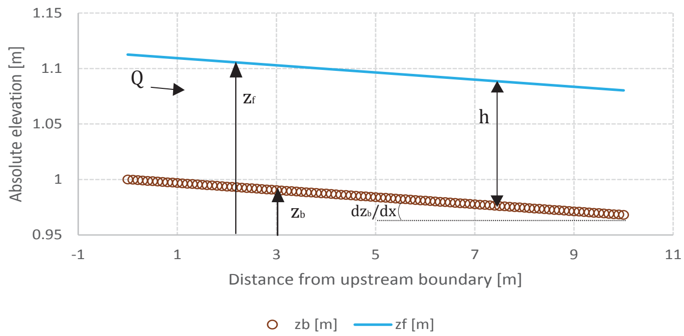


Fig. 1: Sketch illustrating the working-length of the experiments of WC-2003 and the main variables used in the SWE model.

B. Two-dimensional morphodynamics model

Graded sediment processes are successively modeled with the morphodynamic module by: *i*) discretizing the sediment mixture into sediment fractions, where the representative diameter of the i^{th} size class of sediment is user defined, *ii*) the application of a bedload transport capacity formula for each separate fraction and *iii*) using a mass conservation equation adapted for bedload transport for each fraction. The present model assumes a unique layer for the transport of sediment, and the recirculation of sediment was implemented to inject sediments exiting the downstream boundary through the upstream boundary. In this work, the equations to be solved are listed below:

a) Mass balance equation:

The riverbed evolution is computed from the Exner sediment mass balance equation:

$$\partial_t z_b + \frac{1}{\epsilon_0}\nabla\cdot q_b = 0 \quad , \tag{3}$$

where q_b is the volumetric bedload solid discharge per unit of width [m^2/s] and $\epsilon_0 = (1 - P_0)$ with $P_0 = 0.4$ the bed porosity.

The Exner equation can be generalized for graded sediment as [7]:

$$\partial_t z_b + \frac{1}{\epsilon_0}\nabla\cdot\sum_{i=1}^{N} q_{b,i} = 0 \quad , \tag{4}$$

where N is the number of size classes of sediment and $q_{b,i}$ corresponds to the fractional volumetric bedload solid discharge per unit of width of the i^{th} size class [m^2/s].

b) Estimation of sediment transport capacity:

The fractional transport rate $q_{b,i}$ is then estimated using two distinct bedload formulas: *i*) the formula of Meyer-Peter and Müller (MPM) [10] and *ii*) the formula of WC-2003 [23].

The formula of MPM [10] gives:

$$\frac{q_{b,i}}{\sqrt{g\Delta_s d_m^3}} = \alpha\Big(\theta - \theta_c\Big)^\gamma, \text{ with } \alpha = 8, \gamma = \frac{3}{2}, \text{ and } \theta_c = 0.047 \quad (5)$$

where d_m denotes the mean grain size diameter [m], θ the dimensionless Shields parameter and θ_c the dimensionless critical Shields parameter. The calibration of MPM formula on α and γ coefficients lead to other formulations, tested and verified for river applications [25].

The formula of WC-2003 is based on the estimation of the transport rate per unit of width for the i^{th} size fraction $q_{b,i}$ by the relation:

$$W_i{}^* = f(\tau/\tau_{ri}) = \frac{\Delta_s g q_{b,i}}{F_i u_*{}^3} \quad , \qquad (6)$$

where $W_i{}^*$ denotes the form of similarity collapse over fractional transport rate (also refered to as the dimensionless transport rate for the i^{th} class of sediment), τ [Pa] is the bed shear stress, τ_{ri} [Pa] the reference shear stress of the i^{th} size class (also refered to as the similarity parameter), F_i is the proportion of size i on the bed surface, $\Delta_s = \rho_s/\rho - 1$ is the relative submerged sediment density and u_* [m/s] the shear velocity. τ_{ri} is defined as the value of τ at which $W_i{}^*$ is equal to a small reference value of 0.002 [7], [12]. The transport function is defined as follows:

$$W_i{}^* = \begin{cases} 0.002\Phi^{7.5} & \text{for } \Phi < 1.35 \\ 14\Big(1 - \dfrac{0.894}{\Phi^{0.5}}\Big)^{4.5} & \text{for } \Phi \geq 1.35 \end{cases} \quad , \qquad (7)$$

where $\Phi = \dfrac{\tau}{\tau_{ri}}$ corresponds to the ratio between the fluid shear stress over the reference shear stress of size fraction i. The relationship between the variables and processes accounting for the transport of graded sediment are showed in Figure 2.

The definition of the hiding function was made in two steps. First, the authors introduced a hiding function analogous to that used in previous graded transport models [5] [16] [12] in the way that sediment transport rates are lowered for finer fractions (*i.e.* decrease of τ_{ri}) and increased for coarsest material (*i.e.* increase of τ_{ri}):

$$\frac{\tau_{ri}}{\tau_{rm}} = \left(\frac{D_i}{D_{sm}}\right)^b \quad \text{with} \quad b = \frac{0.67}{1 + \exp\left(1.5 - \dfrac{D_i}{D_{sm}}\right)} \quad , \qquad (8)$$

where τ_{rm} is the reference shear stress of the mean size of bed surface, D_i is the grain size of fraction i and D_{sm} the mean grain size of bed surface. In this study, it is important to mention that d_m and D_{sm} are equivalent, as we use a single layer to model surface sediment transport. Secondly, the hiding function was modified to predict τ_{rm} in function of the dimensionless reference shear stress of mean size of bed surface $\tau^*{}_{rm}$:

$$\tau^*{}_{rm} = \frac{\tau_{rm}}{(s-1)\rho g D_{sm}} \quad , \qquad (9)$$

with ρ the water density. The dimensionless reference shear stress $\tau^*{}_{rm}$ was shown to decrease exponentially in function of the sand fraction at the bed surface denoted F_s [7] (wrongly mentionned as the percentage of sand in the original article of Wilcock and Crowe [23]):

$$\tau^*{}_{rm} = 0.021 + 0.015\exp[-20F_s] \quad . \qquad (10)$$

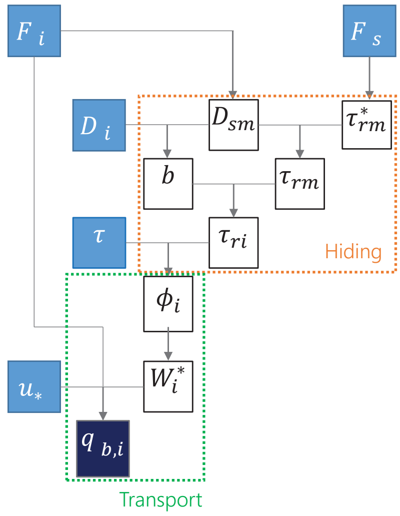


Fig. 2: Scheme of application of the mixed-size sediment transport model of WC-2003. Parameters in blue boxes are input parameters, those in white boxes are intermediary variables computed to estimate the transport rate of size fraction i in the black box.

In the module Sisyphe of TMS, the model was implemented in a new subroutine named `Wilcock_Crowe_bedload.f` which is called by `bedload_formula.f`. At each computational time step, the fraction of sand F_s is computed at each node in the subroutine `bedload_main.f` and transfered to several of its subroutines. The model requires information on the geometric mean grain size of the bed surface D_{sm} which is computed in the subroutine `mean_grain_size.f` (*cf.* Appendix A).

C. Wilcock and Crowe experiments

A brief description of the Wilcock and Crowe [23] experiments is given below. The experiments of WC-2003 were run in a laboratory tilting flume of 0.6 m wide and 8 m long working section, with a 1 m long upstream section dedicated to flow and morphodynamical adaptation. The flow depth was held in a narrow range for all flume runs with values between 0.09 m and 0.12 m, so that we can neglect the viscosity forces. Each experiment was run at least for 60 min to ensure the morphodynamic equilibrium and the data provided by WC-2003 are recorded from the final state of the experiments (*cf.* Tab.I).

The sediment transport model of WC-2003 was calibrated using five GSD by adding different amounts of sand to a gravel mixture. The sand ranged in size from 0.21 to 2.0 mm and the gravel from 2.0 to 64 mm. For each single GSD, the fraction of sand F_s varied from 6.2% to 34.3%, which allows the representation of a wide range of natural gravel-bedded rivers. Sediment transport rates varied at least four orders of magnitude for each mixture, ranging from $1.8{\times}10^{-5}$ kg m^{-1} s^{-1} to $1.2{\times}10^{-1}$ kg m^{-1} s^{-1} [23]. The observations made by WC-2003 [23] assume that particles are subject mainly to bedload transport. The dataset includes hydraulics measurements (water depth, longitudinal flow velocity, discharge) and sedimentary records (transport rate, bulk and surface GSD). More details about the experimental set-up and methods of data recording is documented in Wilcock and McArdell [24].

Among the 49 experimental tests performed by WC-2003, 2 distinct cases (namely BOMC2 and BOMC4) are considered to carry out the numerical investigations presented later. Both laboratory experiments started with the same initial bed surface GSD composed of the bulk-mix denoted BOMC (Bed Of Many Colors, see Fig.3). Different input flowrates are injected in both experiments, resulting in a different final state equilibrium for the geometry (longitudinal slope), hydraulics (*i.e.* water depth, velocity, shear velocity, friction forces, *etc.*, see Tab.I) and sedimentary properties (*i.e.* transport rate, surface GSD, *etc.*, see Fig.3) between both systems.

Name	Q [m$^2/s$]	h [m]	-$\partial_x z_b$	u [m/s]	q_s [g/m/s]	Froude
BOMC2	0.067	0.112	0.0032	0.60	7.1	0.57
BOMC4	0.081	0.094	0.0077	0.90	157	0.90

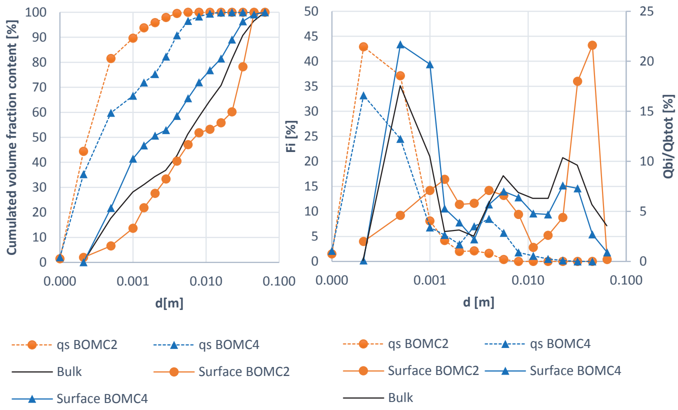
TABLE I: Mean geometric, hydraulic and sedimentary variables recorded in the end of the experiments BOMC2 and BOMC4 of WC-2003.

The choice of reproducing numerically the laboratory experiments using the BOMC GSD was made in order to investigate the behavior of two distinct sediment transport capacity formulas, using extreme conditions of wide size distribution of sediments. Indeed, Fig.3 clearly shows the bimodality (close to trimodality) of the BOMC GSD. The comparison between BOMC2 and BOMC4 experiments allows the study of the system under the *partial* and *full* transport respectively.

III. Numerical modelling of graded sediment transport

The main caracteristics of the numerical models which have been developed to reproduce the two distincts scenarios of the experiments of WC-2003 (BOMC2 and BOMC4 respectively) are presented in this section. A description of the model geometry and the choice of boundary conditions is firstly given. Secondly, a step of calibration on bottom friction forces was necessary to achieve hydraulics uniform and permanent regimes. Finally, further details are presented on the set-up of the morphodynamic models.

The present study is intended to focus on the sediment transport capacity estimation. Other phenomenogical aspects such as riverbed evolution are not discussed here. As a consequence, the present numerical models are designed to start directly from the equilibrium conditions of WC-2003 experiments given in Tab.I.



(a) (b)

Fig. 3: (a) GSD of the bulk, surface and transport for BOMC2 and BOMC4 experiments of WC-2003, (b) Frequency distribution of the surface granulometry and volume fraction of transport of each class of sediment.

A. Mesh and topography

The domain is represented by a computational mesh of 10 m long and 0.6m wide, composed of 4427 irregular triangles with an approximate size of 4 cm, so that the cross-sections are defined by approximately 18 nodes. The initial longitudinal slopes of BOMC2 and BOMC4 numerical experiments are given in Tab.I. Morphodynamics simulations are run using a constant computational time-step equal to 0.01 s. The strategy of model calibration is presented later in the paper.

B. Model boundary conditions

Numerical simulations are run under subcritical flow conditions (Tab.I). To achieve properly the uniform-permanent equilibrium state observed by WC-2003, the upstream boundary condition is defined as a constant discharge Q and the downstream boundary condition is imposed as a constant free surface profile (Fig.1). Friction forces due to the clear sidewalls [24] are neglected in the model. The hotstart generation (uniform steady flow) is reached after 100 s for both experiments.

The upstream boundary condition is set as a recirculating flux of sediment (*i.e.* outgoing sediments are reinjected upstream) generalized for graded sediment transport and the downstream boundary nodes are set as non-erodable (*i.e.* nodes elevation is fixed). Implementation of the recirculation of sediment required the modification of several Sisyphe subroutines (*cf.* Appendix B).

C. Model calibration

The calibration of the hydrodynamic model is performed on the basis of the modification of the roughness coefficient of Strickler (Eq.2). Sensibility analysis are firstly carried out using a small numerical time-step equal to $\Delta t = 0.005$ s with a Strickler coefficient in the range of [40-50] m$^{1/3}$/s. The arbitrary value of $\Delta t = 0.005$ s is chosen to achieve a CFL number equal in the order of 0.125. Results from calibration on water depths and longitudinal velocity are shown in Figure 4 and the absolute error plots suggest that a value of $K = 45$ m$^{1/3}$/s yields satisfactory values of water depths and velocities, which remain less than 1% along the whole longitudinal profile.

The influence of the computational time-step is investigated to determine a good compromise between results quality and simulation time. Table II shows the absolute averaged error per node in water depth [m] using three higher values of Δt equal to 0.01, 0.02 and 0.05 s respectively, using $\Delta t = 0.005$ s as a reference. A time step equal to $\Delta t = 0.01$ s has been chosen for the numerical simulations reported here. Indeed, this value of time-step gives satisfactory results in matter of water depths compared to the reference time-step, whereas the value of $\Delta t = 0.2$ s gives values diverging from the reference with a value over 10%.

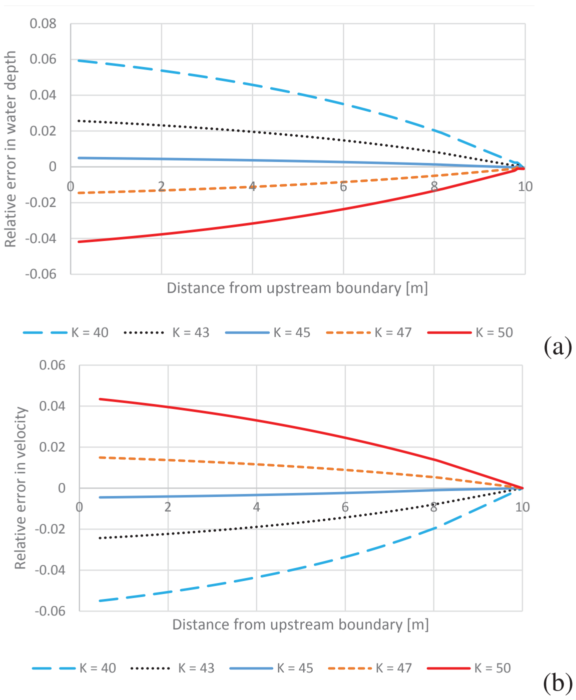


Fig. 4: Relative errors computed along the longitudinal profile for (a) the water depths and (b) the velocities, issued from variation of the Strickler friction coefficient.

Time step [s]	Error in water depth estimation [m]
0.01	$2.3{\cdot}10^{-8}$
0.02	$1.44{\cdot}10^{-3}$
0.05	0.115

TABLE II: Absolute averaged error per node in water depth [m] for different computational time-steps.

D. *Morphodynamic parametrisation*

As a single layer of transport is used in the current model, no GSD evolution of the riverbed is intended to be observed in the numerical experiments. Instead, the numerical models are implemented to run short-term simulations of 100 s, with the assumption that the morphodynamic equilibrium is already achieved from the beginning until the end of the computation. In the experiments of WC-2003, the bed does not present transverse slopes so that bed slopes effects acting sediment transport can be neglected as long as the formation of helical flows.

The discretization of the GSD into size classes of sediment is made in different ways, and the impact of such method of discretization is investigated in the following section:

- Fractional: The GSD is divided into N classes of sediment, where F_{i+1}=F_i $\forall i \in [1 : N-1]$.
- Diametral: The GSD is divided into N classes of sediment, where $D_{i+1} = D_i + A$ $\forall i \in [1 : N-1]$, where A is constant.
- Power-P-Diametral: The GSD is divided into N classes of sediment, where $D_{i+1} = D_i^P$ $\forall i \in [1 : N-1]$, where P is constant.

An example of GSD discretization using the three methods and different numbers of size classes is given in Figure 5. In this work, the following discretizations are considered: fractional for 2, 5 and 10 classes; diametral for 5 and 10 classes; and power-4 for 5 classes and power-2 for 10 classes.

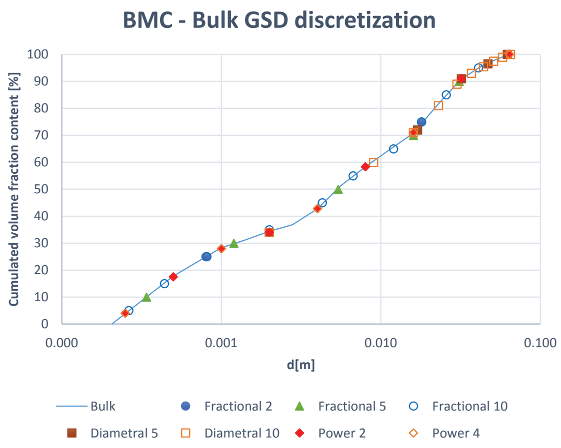


Fig. 5: Discretization of the BOMC GSD using the fractional, diametral and power methods.

IV. Analysis of results and comparison with experiments

Results from numerical investigations are presented hereinafter and are compared with laboratory data from WC-2003. Account is given on several aspects: *i*) the comparison between the MPM and the WC-2003 formulas, *ii*) the impact of GSD discretizing method on sediment transport modelling and *iii*) the influence of initial GSD in the simulations.

A. *Comparison between MPM and WC-2003 formulas*

The choice of the sediment transport capacity formula has a strong impact on the estimation of the bedload transport rate and fractional transport rates in the numerical models (Fig.6&7). While the formulation of MPM often shows discrepancies with laboratory data, the numerical application of the formulation of WC-2003 gives more satisfactory results (as the formuma was determined from the laboratory data)

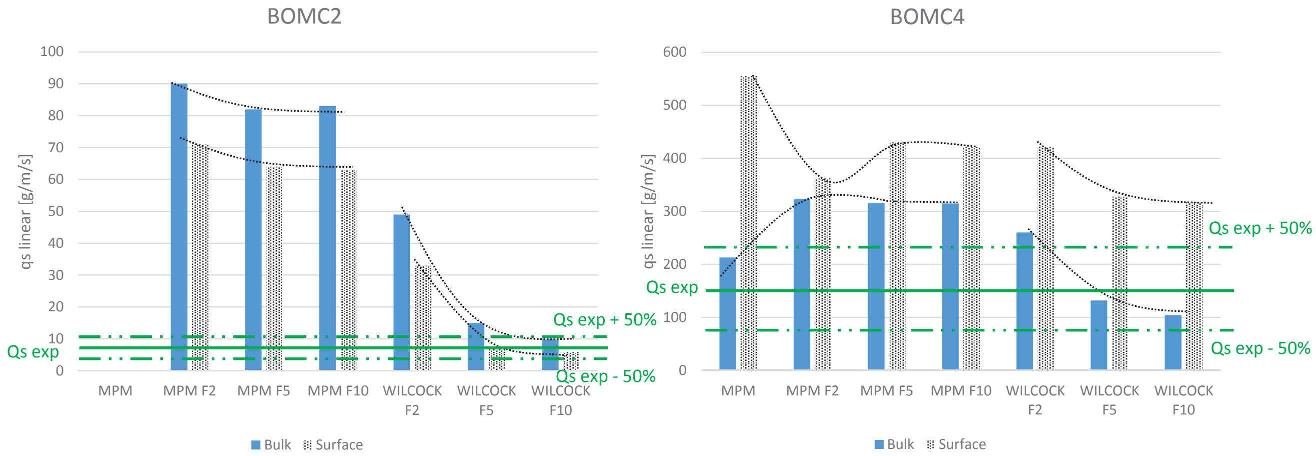
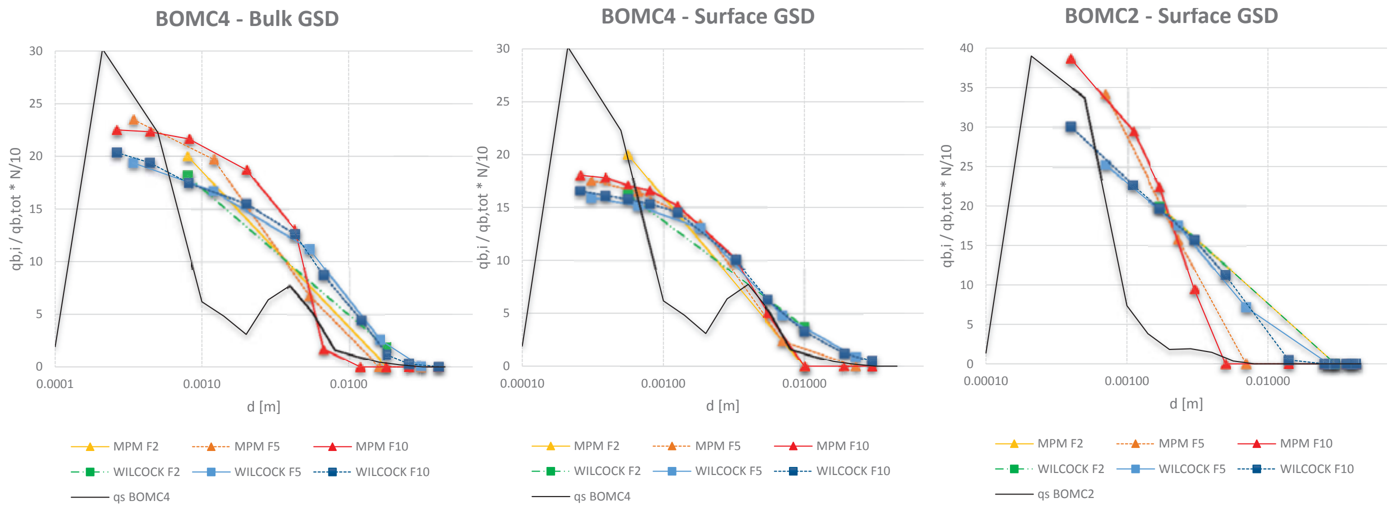


Fig. 6: Comparison between sediment transport rates [g/m/s] computed with MPM and WC-2003 formulas using the fractional discretization method. $Q_{s\ exp}$ corresponds to the flux rates measured in laboratory. F = Fractional discretization and the number stands for the number of size classes of sediment. Black dotted lines help to visualize the convergence.



(a) (b) (c)

Fig. 7: Fractional transport rates of each sediment size-class divided by the number of classes, for BOMC4 experiment with (a) the bulk GSD and (b) surface GSD and the BOMC2 experiment with surface GSD (c).

and physical relevant results. Here, comparisons between bedload formulas are made by using the fractional discretization method, which gives the best results compared with the other discretizing methods (*cf.* IV-B). A convergence analysis on the number of discretizing sediment size classes has been lead to determine their impact on the transport of graded sediment. Convergence of the bedload transport rate (Fig.6) and of fractional transport rates (Fig.7) is observed in every case. It has to be noted that the formula of MPM converges relatively faster than the formula of WC-2003 (Fig.6).

Figure 6 clearly depicts the discrepancies between the sediment transport rates estimated by MPM and WC-2003. Under partial transport mode conditions (*i.e.* BOMC2 experiments), sediment fluxes computed by WC-2003 converge toward a value of 9.8 g.L.s^{-1} for the bulk GSD (*i.e.*) overestimation of 38%) and 5.8 g.L.s^{-1} for the surface GSD (*i.e.*) underestimation of 18%), that lies in the range of sediment fluxes measured in the experimental flume. Altough under total transport conditions (*i.e.* BOMC4 experiments) the bedload transport rate determined by the formula of WC-2003 differs from the measurements (Fig.6), the sediment transport model still computes realistic values with -34% of error for the bulk GSD and 102% for the surface GSD. This difference between laboratory measurements and numerical results can be explained by the experimental data-fitting of the WC-2003 formula. In the other hand, the formula of MPM generally tends to overestimate sediment transport when several size classes of sediment are used, whereas no general rule can be drawn when unisize sediment is used. The difference remains large considering different modes of transport, results provided by the MPM formula lie in a larger range of error of factor around [+50%;+200%] under partial transport conditions and [-100%;+1000%] under total transport conditions. Figure

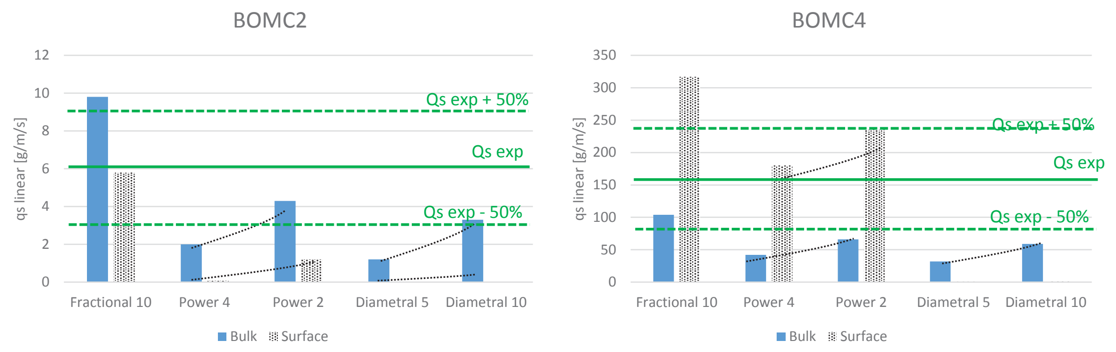


Fig. 8: Comparison between sediment transport rates [g/m/s] computed by the WC-2003 formula using different discretization methods (*cf* Fig.5) presented in IV-B.

7 highlights the differences in the fractional transport rates computed by both formulas. In every case, it is observed the general tendancy of monotonic sediment transport decrease with particle size increase, with the convergence toward a defined GSD shape with the increase of number of size classes. The hiding function of WC-2003 induces a counterbalance in the fractional transport rates, making them decrease for finer particles and increase for coarser ones, in comparison to values determined by the classical formulation of MPM.

While the unimodal shape of the fractional transport rate is reproduced for the BOMC2 experiment, the bimodal shape of sediment transport is not represented by the BOMC4 numerical models (Fig.7). According to the Figure 7, the formula of WC-2003 tends to smooth and skew the sediment transport, so that abrupt transitions conducting to bimodality of transport cannot be accurately represented in the models.

B. Comparison between GSD discretization methods

Natural rivers GSD commonly show a continuous sorting of sediment and a bimodal behavior. The representation of such density functions in the WC-2003 model requires the subdivision of the initial surface GSD into size classes of sediment. Different discretization methods can be used, and their impact on fractional sediment transport is not yet known. To test the impact of the GSD discretization method on fractional sediment transport estimation, several scenarios are proposed where three distinct methods are used (*cf.* III-D).

According to Figures 6&8, diametral and power discretization methods always give lower transport rates than methods based on the fractional discretization. This property can be explained among other things by the fact that the value of the mean diameter D_{sm} is strongly dependent to the chosen method of discretization. Indeed, Figure 9 shows the tendancy of D_{sm} underestimation for the fractional method and overestimation for the other ones, besides the fact that D_{sm} converges with the number of size classes increasing. Methods based on the power and diametral discretization show the same tendancy of total sediment transport increase with the number of discretizing size classes increasing. As a rule, the diametral discretization shows unsatisfactory results, with an error inferior to -50% on fluxes estimation, and a complete cessation of transport using the surface GSD. Altough the power-2-based method gives relatively acceptable values of

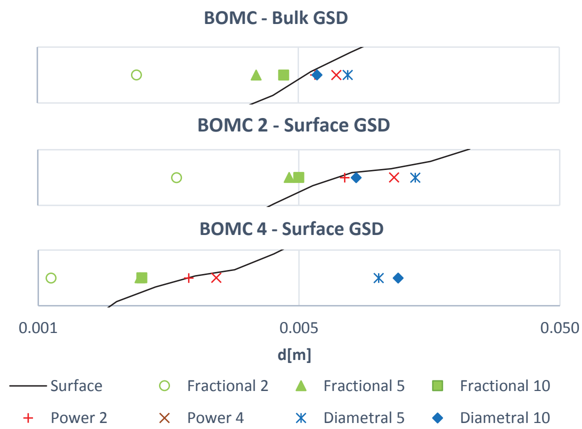


Fig. 9: Computed value of the mean diameter D_{sm} using different GSD discretization methods presented in IV-B.

transport rates (-63% of error for the bulk and +49% for the surface GSD) at full transport conditions (*i.e.* BOMC4 experiment), it tends to underpredict transport (-39% of error for the bulk and -83% for the surface GSD) at partial transport conditions (*i.e.* BOMC2 experiment).

To conclude, the current model needs an appropriate discretization of the GSD to be set-up, and that the fractional method seems to be appropriate to this model.

C. Influence of initial sediment composition

The choice of the surface GSD in the numerical model has a strong impact on the bedload transport rates and on fractional transport rates. Here, attention is given on the results obtained by the WC-2003 formula with *i*) a bed composed of the bulk GSD called BOMC and *ii*) a bed composed of the surface GSD sampled in the end of the experiments of WC-2003.

Firstly, it is obvious that different GSD provide different D_{sm} (Fig.7&9), which will have consequently an impact on the estimation of bedload transport (Eq.7).

Bedload fluxes computed here for 10 size classes of sediment under partial transport conditions (*i.e.* BOMC2) for the surface GSD are lower than the ones computed with the bulk

GSD of a factor equal to 3 (Fig.6). This tendancy is inverted under full transport conditions (*i.e.* BOMC4), where the total transport rate estimated from the surface GSD is stronger than the one produced by the bulk GSD, with a lower factor equal to 1.7.

Figure 7 highlights the differences in estimated fractional transport rates depending on the choice of the riverbed composition for the BOMC4 experiment. While the surface GSD gets finer than the bulk GSD at the end of the BOMC4 laboratory experiment (Fig.5), the comparison between transports rates given by the bulk GSD (Fig.7a) and the surface GSD (Fig.7b) show that a counterbalancing effect is applied to fractional transport rates. Indeed, the distribution of fractional transport rates tends to be finer by using the bulk GSD and coarser using the surface GSD.

V. Conclusion

Numerical investigations based on the experiments of WC-2003 constitute a first step for the modelling alluvial bars morphodynamics. Whereas most of engineering applications still rely on the use of classical sediment transport models (*i.e.* MPM) to model graded sediment processes, the present study outlines the importance of using adapted models such as the one proposed by Wilcock and Crowe [23]. The implementation of such sophisticated models is a necessary condition to improve the estimation of the fractional transport rates in natural rivers, thus a better simulation of bar armoring and armor break-up in the framework of bars modelling. Results from numerical modelling show that the formula of WC-2003 gives more relevant fractional and total transport rates compared to the ones given by the classical formula of MPM.

However, the numerical application of graded sediment transport models is not straightforward, as it shows in this case a lot of sensitivity to *i*) the method of GSD discretization and *ii*) the number of size classes of sediment. On the one hand, the method of fractional discretization shows more satisfactory results than the other methods presented in the paper. On the other hand, previous numerical studies implementing the formula of WC-2003 were conducted with only two size classes of sediments made of sand and gravel respectively [2], while the formula of transport was originally calibrated on 15 size classes of sediment. Moreover, in the configuration of the experiments, it is observed that D_{sm} may vary of several orders of magnitude depending on the number of size classes and that bedload transport rates become relevant starting from a number of 5 size classes of bed material.

This study also underlines the importance of the choice between the surface GSD and the bulk GSD for the modelling of alluvial bars dynamics. If the numerical model is designed to investigate long-term scenarios of bars dynamics starting from a flat bed, the initial surface GSD in the model should be the one of the bulk mix, and the model would require the definition of an active layer to take account of the vertical sorting processes allowing the surface GSD to change in time. If bars are already present as initial topographic conditions in the model, it would be more appropriate to dissociate the substrate and the active layer GSD together with planimetric variations of the GSD, which makes the process of model setting-up manifestly more difficult.

Altough the model of WC-2003 remains a satisfying estimator for the total transport rate, the models seems to be limited for the determination of fractional transport rates. The distribution of fractional transport rates tends to be smoothed out and no bimodality of transport is observed in contrast to the experimental runs. To complete this study, other experimental runs could be reproduced, including a different bulk GSD and greater number of sediment size classes. Further numerical models could be relaxed by including the definition of the active layer concept [8] in order to account for the temporal variation of the surface GSD. Another limitation is based on the fact that a wide GSD is associated to a decrease of bed porosity (which can be represented by the *Appolonian gasket* problem [19]), which require an appropriate formulation to be took into account.

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Appendix A - Mean diameter calculation

Originally, the definition of the average grain size D_{avg} [m] in Sisyphe was based on the simple weigthing average:

$$D_{avg} = \sum_{i=1}^{N} F_i D_i. \tag{11}$$

Where F_i and D_i [m] correspond to the volume fraction content and the diameter of the i size class respectively. In this work, the definition of D_{avg} is replaced by the mean surface diameter D_{sm} [m]. The calculation of D_{sm} was defined by a linear interpolation between size classes of sediments where fractions of sediment are respectively upper and lower 50% of cumulated material.

Appendix B - Implementation of sediment recirculation in Sisyphe

The implementation of sediment recirculation is made in two steps and generalized for transport of mixed-size sediment. Firstly, the variable of transport rate along the boundary nodes denoted `QBOR` is modified in `conlit.f` and `disimp.f` subroutines. As a result, volumes of entering sediments printed by `bilan_sisyphe.f` are equal to exiting volumes of sediments. Secondly, the value of the variable of transport rate in the whole domain `QSCL` which is originally computed with the bedload transport formula is replaced at upstream nodes by the value of the exiting volume of sediment.