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Analysis of 3D-bed form migration rates

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ABSTRACT: Migration rates of 3D-bed forms were analyzed applying cross-correlation techniques to subsequently recorded digital elevation models (DEMs) of polystyrene dune fields. The data were obtained from a scale model of an 8 km long section of the River Oder at the Federal Waterways Engineering and Research Institute (BAW), Germany. The lightweight sediment model was designed to simulate bed-load transport and to investigate different river maintenance strategies. In the experiments, time series of bed form movement were measured with high spatial and temporal resolution using a 3D photogrammetric measurement system. Cross-correlation techniques were used to estimate bulk dune field migration velocities as well as depth resolved migration rates. The latter were obtained using the high spatial and temporal resolution of the data by dividing the measured DEM in sub-sections. This analysis allowed for the detailed investigation of longitudinal and transverse migration rates. The preliminary results presented in the paper suggest that it is possible to relate small scale bed form migration rates to bed form deformation processes.

Keywords: Sediment Transport, Dunes, Morphodynamics, Light-weight sediment, Random field approach

1 INTRODUCTION

Bed forms in river flows determine hydraulic resistance, sediment transport, channel morphodynamics, and hydraulic habitat for biota. They also often present a major problem for engineering structures (e.g., water intakes or discharges, pipelines, groynes, etc.) and may introduce severe restrictions to navigation. In fact, dunes are considered as the most important bed form in practical river engineering (Engelund & Fredsoe, 1982; Southard, 1991) and the prediction of the associated flow and sediment transport still presents a major obstacle in the solution of sedimentation problems in alluvial channels (ASCE, 2002).

When sediment transport takes place over bed forms, their migration rate represents an important kinematic characteristic determining bed load transport (Best, 2005). The corresponding transport rate may be estimated using the so called dune tracking method in which bed load transport rate is related to bed form height and speed (e.g., Engel & Lau, 1980; Jerolmack & Mohrig, 2005).

Until today this method has mostly been applied to 2D longitudinal sections of laboratory and field data and studies focusing on 3D bed forms are rare. However, recent advances in measure-

ment technology (e.g., Henning et al., 2009) make it possible to measure 3D bed form topographies with high spatial and temporal resolution during water flow. Such data enable detailed spatial analyses of bed form roughness and bed form migration.

In spite of these developments, approaches for the determination of bed form geometry as well as migration rates are mostly based on the analysis of 2D-longitudinal profiles. In fact, studies of dunes to date have principally measured and characterised 2D sections of 3D-dune fields, with consequent limited interpretations of the dynamics of natural 3D-dune fields. This shows the need for the development of adequate methods to investigate the 3D-nature and dynamics of bed forms.

Using the random field approach, it becomes possible to describe bed form geometry and dynamics using various statistical measures such as the probability distribution of bed elevations and its moments, space-time correlation and structure functions, and frequency and wave number spectra (e.g., Nikora et al., 1997). Altogether, these measures provide a full, in practical sense, description of geometry and roughness properties due to bed forms. Hence, using the random field approach it becomes possible to investigate the

3D-geometry and dynamics of bed forms. Furthermore, using data with a high spatial and temporal resolution it also becomes possible to investigate the deformation processes of 3D-bed forms (e.g., Nikora et al. 1997, McElroy & Mohrig, 2009).

The present paper addresses the aspect of dune migration and deformation processes using cross correlation analyses of dunes from scale model investigations using polystyrene sediment particles. The findings are compared to dune field bulk velocity estimates.

2 SCALE MODEL & DATA

2.1 Model investigations

This study is based on data from experiments which were carried out at the Federal Waterways Engineering and Research Institute (BAW), Karlsruhe, Germany, in an 80 m long distorted scale model (1:100 horizontal, 1:40 vertical) of the Oder River. The scale model, described in detail in Hentschel (2006), was originally built to investigate navigational issues and covers an 8 km long trained river reach along the German-Polish border (km 654.7 to km 662.5) including a 5 km long straight section and a 3 km long curved section (Figure 1). In this reach, the Oder River is characterized by a highly dynamic morphological behavior due to its fine-gravel/coarse-sand bed material (mean diameter $d_m = 0.92$ mm, $d_{60}/d_{10} = 2.3$) and associated bed forms (dunes and alternate bars). In order to achieve naturelike transport conditions and to guarantee the similitude of grain Reynolds number and dimensionless shear stress, the model was constructed as a lightweight model (e.g., Hughes, 1993) using polystyrene granules with lesser density and coarser diameter but comparable inhomogeneity ($\rho_s = 1055$ kg/m³; $d_m = 2.1$ mm, $d_{60}/d_{10} = 2.0$) as bed material. The polystyrene material was constantly fed during the experiments at the model inflow section.

In this paper, the analysis of data from an experiment carried out with a steady discharge corresponding to the mean annual discharge ($Q = 460$ m³/s in prototype scale) with a mean Froude number of $Fr \approx 0.2$ is presented. The initial bed configuration consisted of bed forms from a previous experiment carried out with a lower discharge. In order to avoid effects arising from the model distortion, the analysis is carried out in model scale.



Figure 1. View of the model and detailed photograph of bed forms and training structures.

2.2 Bed elevation data

Detailed information on bed topography was available from measurements with a 3D-photogrammetric system (see Godding et al., 2003, Henning et al., 2009 for details). The system can be used to measure bed topography during water flow through the water surface with high temporal and spatial resolution. In the experiment, the system was positioned in the straight section of the model and the bed topography of a 3 m long and 2 m wide section was measured with $\Delta t = 10$ s, $\Delta x = \Delta y = 2.5$ cm and vertical precision ± 1 mm. In total, 4000 subsequent digital elevation models were recorded.

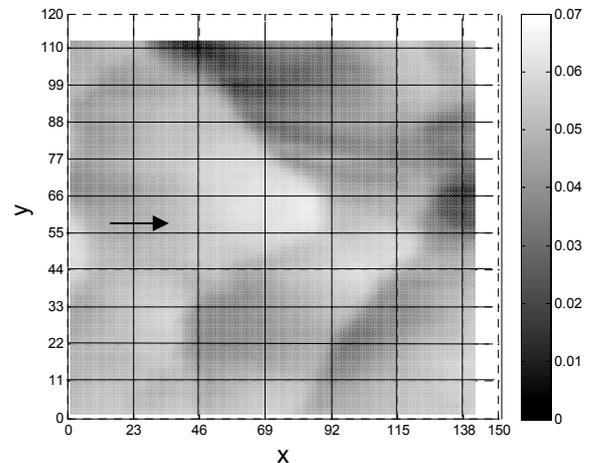


Figure 2. Digital elevation model of the test section measured at $t = 5000$ s. The grid lines indicate sub-sections (see Section 3.2). The units of the colorbar are given in [m] and the units of x and y -axis are given in [cm].

For the analysis, each digital elevation model (DEM) was first rotated around a constant angle so that the flow direction was aligned with the x -axis. Then the DEMs were re-gridded linearly with a resolution of $\Delta x = \Delta y = 0.01$ m as the data points in the x - y plane were not equally spaced. A 1.41 m long and 1.11 m wide section with a grid size of 142 x 112 data points (see Figure 2) was extracted from each re-gridded DEM for the subsequent analysis in order to minimize the influ-

ence of channel boundaries and river training structures such as groynes. We acknowledge that the resolution of the analysed DEMs is finer than the resolution of the measured DEMs. However, for the development and application of the methods presented below it was helpful to increase the horizontal resolution for the cross correlation analysis. In our further experiments and analyses the measurement resolution will be adjusted. Further information on the statistical analysis of the time series may be found in Aberle et al. (2010).

3 ANALYSIS & RESULTS

3.1 Bulk dune field migration

Spatially averaged bed form migration rates were determined using a cross-correlation technique based on the space-time correlation function $R(\Delta x, \Delta y, \Delta t)$, where Δx and Δy define spatial lags and Δt defines the temporal lag (e.g., Nikora et al., 1997). Values of $R(\Delta x, \Delta y, \Delta t)$ were calculated using constant values for Δt and the spatial lags Δx and Δy yielding the maximum ordinate of the correlation function were used to define the average bed form traveling distance in longitudinal and transverse direction, respectively. Variation of Δt yielded $\Delta t = 40$ s as most appropriate temporal resolution in order to resolve the bulk migration rates. Knowing Δx , Δy and Δt , the average longitudinal and transverse migration speeds were calculated according to $u = \Delta x / \Delta t$ and $v = \Delta y / \Delta t$. The given values of $\Delta t = 40$ s, $\Delta x = \Delta y = 0.01$ m resulted in a minimum detectable spatially averaged migration speed of 0.25×10^{-3} m/s.

Figure 3 shows u and v as a function of time and indicates that the bed forms, on average, did not migrate with a constant velocity. In general, 17 different migration rates were observed for u , ranging from 0.75×10^{-3} m/s to 4.75×10^{-3} m/s. It is worth mentioning that Figure 3 also reveals a local maximum and minimum of u at $t \approx 3.2 \times 10^4$ s which was caused by a sudden change in the transport pattern (see Aberle et al., 2010). The reason for this pattern change remains unclear and will be investigated further in upcoming analyses.

The fluctuations in the bulk migration rate can be associated with the irregular 3D-nature of the bed forms, their different sizes, and their deformation within the analysis domain. Similar deformation processes have been reported by, e.g., Jain & Kennedy (1974), Cheong (1992), and Jerolmack & Mohrig (2005). The temporal mean of the longitudinal migration rates corresponds to $\bar{u} = 1.95 \times 10^{-3}$ m/s while the mean of transverse velocities is reasonably close to zero ($\bar{v} = 0.14 \times 10^{-3}$ m/s).

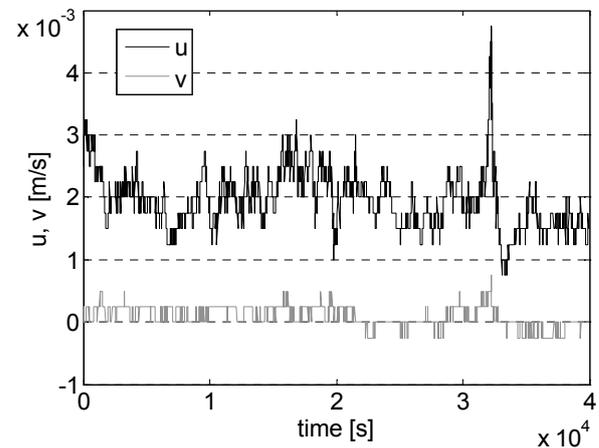


Figure 3. Dune field migration rates as functions of time.

In order to investigate the relationship between u and bed form size, the standard deviation of bed elevations σ_d determined from each individual DEM as a surrogate measure for bed form height was used (e.g., Nordin, 1971; Nikora et al. 1997). The sample size of an individual DEM is not large enough to obtain stationary conditions in a statistical sense (see Aberle et al., 2009) and it is therefore not possible to conclude on the ensemble standard deviation from these values. Nonetheless, the standard deviations derived from each DEM may be interpreted as indicators for instantaneous bed form height as they represent the average fluctuation of bed elevations around the corresponding mean bed level. Furthermore, the same effect impacts the application of the cross-correlation function. Hence, the absence of stationary conditions may contribute to the observed scatter in Figure 3.

Figure 4 shows the relationship between σ_d and bulk migration rate u . In this figure, the standard deviation represents the mean value of the σ_d values estimated for identical migration rates (see Figure 3). The data indicate that σ_d and hence the magnitude of bed form height decreases with increasing migration rate. This observation is in agreement with reported observations that small dunes travel faster than larger ones for identical hydraulic conditions (e.g. Kostaschuk & Ilersich, 1995 Coleman & Melville, 1996). Note that values for $u < 1.0 \times 10^{-3}$ m/s and $u > 3.25 \times 10^{-3}$ m/s are associated with the aforementioned event at $t \approx 3.2 \times 10^4$ s (see Figure 3).

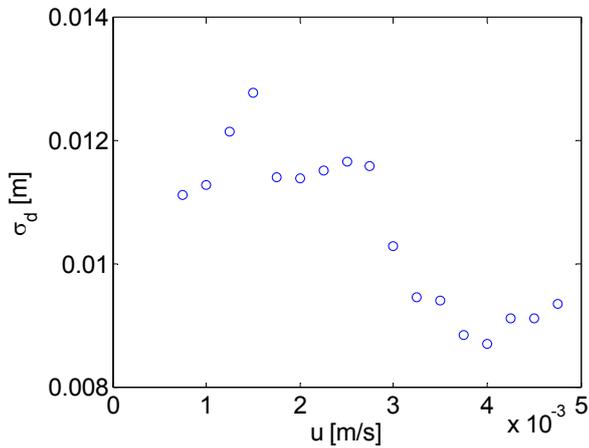


Figure 4. Dune field velocities u and standard deviations σ_d .

3.2 Depth-resolved dune field migration

In the experiments, it was also visually observed that individual bed forms moved faster than the bulk dune field. In order to investigate this issue in more detail, the cross-correlation analysis was repeated with a higher spatial resolution. For this purpose, each DEM was subdivided in 6 longitudinal and 10 transverse sub-sections with the dimension of 0.22 m in longitudinal and 0.10 m in lateral direction (see Figure 2). Each subgrid consisted of 23×11 data points leading, when put together to the larger DEM, to a slightly smaller analysis area than the one used for the bulk analysis. Furthermore, due to the use of sub-sections the time interval could be reduced to $\Delta t = 10$ s in this analysis, the spatial resolution, as in the bulk analysis, is $\Delta x, \Delta y = 0.01$ m. Figure 5 exemplarily shows two subsequent sub-section DEMs.

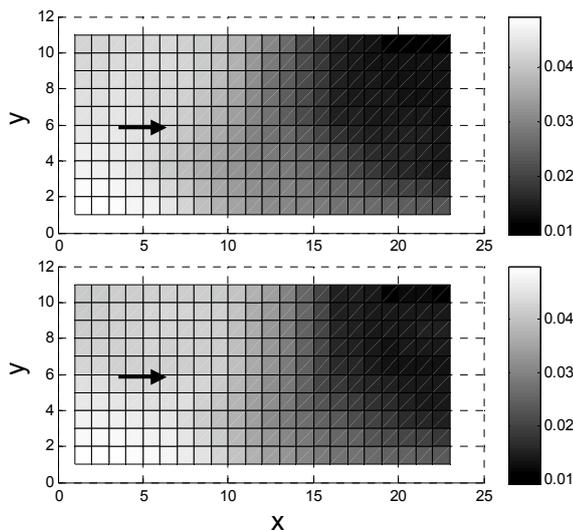


Figure 5. Sub-section DEMs at $t = 5000$ s (top) and $t = 5010$ s (bottom), colorbar units are given in [m] and the units of x and y -axis are given in [cm].

For each DEM and sub-section, the average bed elevation h was calculated followed by the determination of the longitudinal and transverse mi-

gration velocities u and v . Figure 6 shows the probability distribution of the longitudinal migration velocities, and indicates that the longitudinal migration rates were approximately log-normally distributed. A similar result has been reported by Cheong (1992).

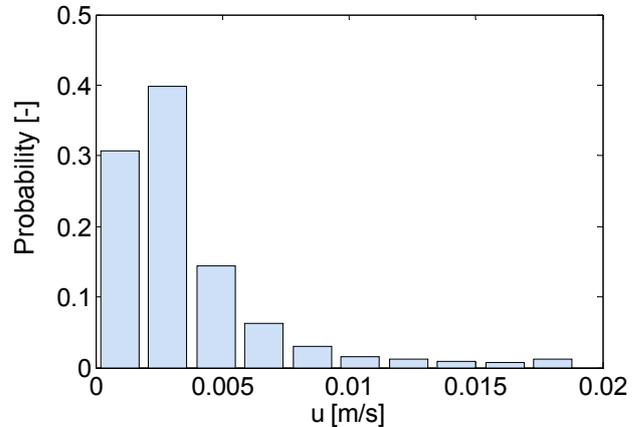


Figure 6. Probability distribution of longitudinal dune velocities.

In order to determine the dependency of u and v on h , the available data triple $\{h, u, v\}$ were sorted by h in ascending order and classified into 50 bins with variable size so that each bin contained 4798 data points. Following this classification, the mean migration rates $\langle u \rangle$ and $\langle v \rangle$ as well as the corresponding average bed elevation $\langle h \rangle$ were calculated for each bin. This methodology can be used to investigate the vertical distribution of the averaged bed form migration rates as shown in Figure 7. By applying the above procedure instead of classifying the data into equidistant height containers the distortion of the velocity profile at the margins of the height distribution, where due to sparse data $\langle u \rangle$ and $\langle v \rangle$ are nonstationary, was avoided.

In general, it was found that $\langle v \rangle$ were close to zero (see Figure 7). In order to resolve the magnitude of the transverse migration velocities we decided to display the mean absolute values $\langle |v| \rangle$ in Figure 7. Furthermore, the minimum $\langle h \rangle$ has been scaled to zero, with the maximum $\langle h \rangle$ then being 0.0835 m (the maximum ordinate).

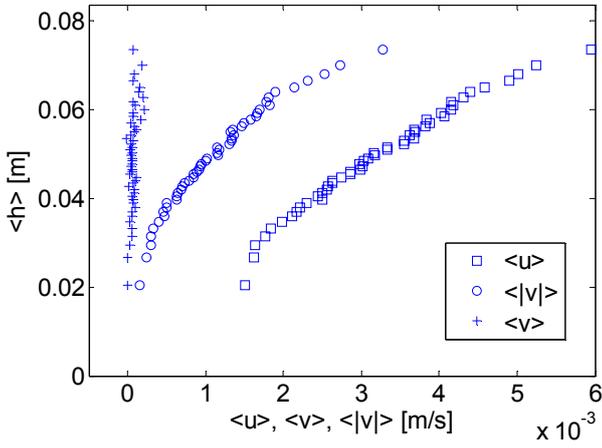


Figure 7. Vertical dune field velocity profiles.

According to Figure 7, both $\langle u \rangle$ and $\langle |v| \rangle$ increase with $\langle h \rangle$ and the shape of the distribution suggests that both migration rate distributions may be approximated by a power law. This is shown in Figure 8, where the data are re-plotted in log-log scale and the migration rates are normalized using the arithmetic mean $u_m = 3.24 \times 10^{-3}$ m/s and $v_m = 1.16 \times 10^{-3}$ m/s for the longitudinal and absolute transverse velocities, respectively. The depths $\langle h \rangle$ are scaled with the average bed elevation $h_m = 0.049$ m. The figure reveals that the longitudinal migration rates follow a power law only a certain distance above the troughs while the absolute values of the transverse migration rate follow a power law almost over all bed elevations.

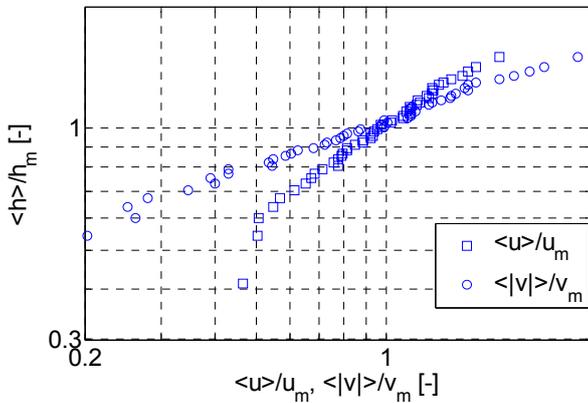


Figure 8. Power-law relations between $\langle u \rangle / u_m$, $|v| / v_m$ and $\langle h \rangle / h_m$.

It is also interesting to note that the variability of the migration rates increases with increasing distance above the troughs. This can be shown by relating the standard deviation of the longitudinal migration rates σ_u obtained for each bin to the distance above the troughs (Figure 9). The figure indicates that σ_u may be a linear function of $\langle h \rangle / h_m$.

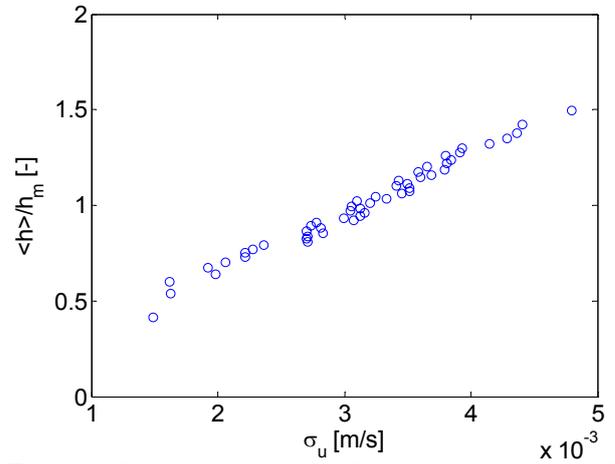


Figure 9. Standard deviation of longitudinal dune field velocities as a function of height.

4 DISCUSSION

In this paper, bed form migration rates were investigated using different spatial resolutions. The bulk analysis presented in section 3.1 resulted in an average longitudinal migration rate of $\bar{u} = 1.95 \times 10^{-3}$ m/s and an average transverse migration rate of $|\bar{v}| = 0.14 \times 10^{-3}$ m/s. These bulk migration rates are lower than the arithmetic means of $u_m = 3.24 \times 10^{-3}$ m/s and $v_m = 1.16 \times 10^{-3}$ m/s obtained from the sub-section analysis. However, it is interesting to note that u from the bulk analysis is similar to the lowest observed velocity in the depth-resolved analysis, which corresponds to the speed of the bed form troughs (see Figure 7). The relatively low migration rates obtained from the bulk approach indicate that smaller bed forms are not identified by this methodology and that the correlation between two subsequent bed scans is, for the bulk analysis, governed by large scale structures rather than by small scale structures.

This is also reflected by Figure 10 showing the comparison between the bulk analysis migration rates and averaged migration rates of the sub-section approach. In this plot, velocities $\langle u \rangle$ from all individual sub-sections for each time-step were summarized in a single mean, so that both values represent the same parameter. Both curves in Figure 10 follow the same pattern although the sub-section rates are more fluctuating and are always larger than the bulk migration rates. This result obviously has significant implications for the dune tracking method which requires detailed information on bed form migration rates.

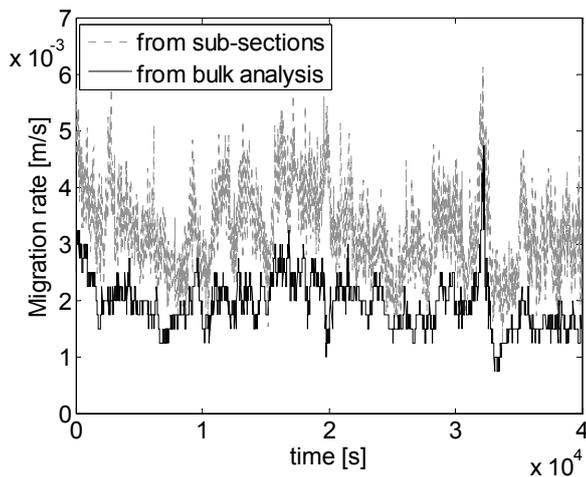


Figure 10. Comparison of spatially averaged longitudinal dune field velocities for the sub-section analysis and the bulk analysis.

The results from the bulk analysis did also not allow for the unambiguous identification of bed deformation processes. Using the cross-correlation technique, the only variable indicating bed deformation processes is the correlation coefficient. In general, correlation coefficients somewhat lower than unity have been observed in the analysis indicating that the bed forms did deform during migration (see also McElroy & Mohrig, 2009). However, the correlation coefficients were still very high and hence they are not suitable to quantify bed deformation. Instead, the results show that such an assessment becomes possible from the analysis of the migration rates obtained from the sub-section analysis.

This analysis revealed a depth dependency of the longitudinal migration rates. The finding of migration rates increasing with increasing distance to the troughs is a strong indicator for bed form deformation processes. In fact, for form stable bed forms one would expect only a single migration velocity over the depth. However, as the migration rate is varying with depth for the present data, bed form movement must be associated with deformation processes. In this context it is interesting to note that the absolute values of the transverse migration rates indicate a strong transverse deformation although, on average, the transverse migration rate is close to zero.

Deformation processes are associated with continuous transformation of bed forms through merging and separation and are an important issue for 3D bed form migration. This can be brought into context with the variance cascade model developed by Jain & Kennedy (1974). According to this model, bed forms are continuously generated at larger wave numbers. Small bed forms travel faster than larger ones and thus will overtake the larger structures. However, the bed forms cannot pass through each other and hence the shorter bed

forms are absorbed by the longer structures. As a consequence, new bed forms form which are longer and higher than the initial merging bed forms. This process continues until the bed forms reach a limiting size and cannot grow further or so long that they are, for higher wavelengths, attenuated by the flow. This physical process may be one reason for the observed variability of the migration rates over depth. Our further analyses will focus in more detail on this issue, also taking into account spectral analysis of both areal scans and time series (see Hino 1968, Jain & Kennedy 1974, Nikora et al., 1997 for further information).

In the literature, it has also been found that deformation processes can be influenced by saltating particles jumping from the crest of the dune to the next downstream bed form (e.g. McElroy & Mohrig, 2009). The amount of saltating particles depends on the intensity of flow and may, under field conditions, be up to 60 % (Mohrig & Smith, 1996) or even up to 75 % as reported by Simons et al. (1965). However, in the current study such effects have not been observed and are therefore not significant. The same applies to grain sorting processes and potential effects due to unsteady flow (hysteresis).

On the other hand, the occurrence of alternate bars may have an effect on the observed bed form velocity profiles. However, the analyzed area was significantly smaller than the length of the observed alternate bars (4 to 8 m) and hence the influence of these bed features may be, in general, negligible. However, they may be responsible for the observed peak in the migration rates at $t \approx 3.2 \times 10^4$ s (Figure 3). This will be further investigated in follow up studies. It is also worth mentioning that the influences of groyne head scours were minimized by extracting an area from the middle of the channel. To what degree such structures may bias the results presented in this study remains an open question.

5 SUMMARY & CONCLUSIONS

In this paper, we present the analysis of 3D dune movement using time series of digital elevation models obtained from a scale model of a natural river. Bed form migration rates were obtained by applying a cross-correlation method at different spatial scales. The presented method was successfully applied to resolve vertical variations in bed-form migration rate.

The analysis revealed an increase of the longitudinal and absolute transverse migration rates with increasing distance to the minimum bed level which was attributed to bed form deformation processes. It was also found that trough regions

were travelling with lower speeds than crest regions and that the variability of migration rates increased with distance to bed form troughs. Average migration rates were distinctively higher for the depth resolved sub-section analysis than for the bulk analysis. The latter is much less influenced by deformation and yields an average migration rate being close to the trough velocities as estimated from the sub-section analysis.

The preliminary results presented in this paper provide valuable information for the analysis of 3D-bed form movement and as a consequence for the dune tracking method. However, further research is also required for the assessment of 3D-bed form height and length so that bed load may be estimated reliably using the dune tracking method.

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