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An Analysis of MLR and NLP for Use in River Flood Routing and Comparison with the Muskingum Method

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ABSTRACT: The development of precise and simple methods for flood simulation has greatly reduced financial damages and life losses in many flood-prone regions of the world. Most of the flood simulation techniques and procedures implemented up-to-date are based on the Saint-Venant's one-dimensional equation governing unsteady flows. In the present study, two new approaches for tackling the problem of optimal calibration of a flood model have been introduced. The first method is based on nonlinear programming (NLP), which permits to determine the optimum values of the routing coefficients in the diffusion wave or Muskingum method by minimizing a misfit function under the constraint of satisfying the continuity equation. The second method is based on Multiple Linear Regression (MLR) of in- and output variables in the Muskingum equations, which allows the direct computation of the routing coefficients. To calibrate and verify the two new routing models as well as of the traditional Muskingum method three (one for calibration and two for verification) observed flood hydrographs in a limited reach of Mehranrood River in northwest Iran are used. The results obtained by these two new methods are compared with those of the classical Muskingum method. It is found that the NLP- and the MLR- routed hydrographs come as close, if not better, to the observed output hydrographs as those of the Muskingum method. This is also corroborated by similar high values for the coefficient of determination R2 of the adjustment of the simu-lated to observed hydrographs for the three routing methods. However, limitations of all three kinematic-wave type routing methods become clear during the verification routing simulation for one flood even with a sharply rising input hydrograph, in the case of which, the application of full dynamic wave routing gives much better results. In spite of these restrictions - typical for kinematic wave routing methods - the two new parameter optimization methods proposed here for the automatic calibration of the routing coef-ficients in the widely used Muskingum method are powerful and reliable procedures for flood routing in rivers, not to the least due to the fact that they are convenient to use

Keywords: Flood wave routing, Muskingum, Nonlinear programming, Multiple linear regression

1 INTRODUCTION

Nowadays, the occurrence of floods all over the world has resulted in tremendous economic damages and life losses. Thus, the correct prediction of the rise and fall of a flood, i.e. flood wave routing, is significant importance. Although the use of numerical simulation methods makes the prediction of this complex hydraulic phenomenon feasible, many fallacies in doing this properly still exist. The fundamental differential equation to describe one-dimensional unsteady river flow is the Saint-Venant equation (Chow *et al.*, 1989), which is basically a special form of the Navier-Stokes applied to an inclined section of an open channel, where internal viscosity-induced frictional forces within the fluid are neglected against shear stresses induced by bed-friction or wind forces. This equation is to be solved in conjunction with the continuity equation for a control volume of water within the channel. Because of the nonlinearity of the convective acceleration term in the Saint-Venant equation, in its most complete form it can only be solved numerically at some non-negligible costs. For this reason, alternative approaches for flood wave routing, known under the names of diffusion and kinematic wave method - which are easily derived from the full Saint-Venant equations (also called the full dynamic wave method) by dropping the acceleration term (diffusion wave method) or both the acceleration- and the pressure term- (kinematic wave method) - have been proposed and which nowadays are widely used in practice.

Diffusive wave theory was firstly presented by Hayami (1951). By simplification of the momentum equation and introduction a linear diffusion coefficient, Hayami (1951) derived an advection-diffusion equation that he solved analytically. An analysis of the kinematic wave routing theory was made later by Lighthill and Whitham (1955) who showed that the main part of a flood wave is approximated by a kinematic wave traveling downstream, whereas the part arising from the full solution of the Saint-Venant Eq., i.e. the dynamic wave, makes up only a small portion of the flood body, but travels in both upstream and downstream direction relative to the crest of the kinematic wave. Thus it is clear that kinematic wave theory cannot model backwater-effects. There have been a lot of investigations since then to what extent the various simplifications in the Saint-Venant Eqs. are valid for routing in a particular channel (e.g. Ponce *et al.*,1978; Weinmann and Laurenson, 1979; Ferrick, 1985). These authors found, among other things, that the kinematic wave approximation is valid for moderately steep channels.

The numerical implementation of the kinematic wave approximation is usually the Muskingum- and/or the Muskingum-Cunge method (Cunge, 1969; Chow *et al*., 1989), where the latter has been shown to have, in addition, some diffusion-wave type properties, although the method is not directly derived from the Saint Venant equations itself (see Section 2). In spite of this inherent limitation of the Muskingum method, in addition to the problem of the proper specification of some heuristic routing coefficients for a particular stream reach (see Section 2), this method has been, since its inception, because of its computational expediency, the method of choice in most flood-routing applications, particularly for real-time forecasting (Barbetta *et al.*, 2011; Perumal *et al.,* 2011). It is thus of no surprise that numerous hydraulic research of the last decades has been devoted to the problem of how to determine the proper routing coefficients in the Muskingum method, either by using information on stream channel characteristics and/or by some kind of calibration of observed flood hydrographs.

One of the first authors to determine parameters in the Muskingum method was Gill (1978) who used linear least squares to determine the two unknown parameters in the prism/wedge storage term which is the basis of the Muskingum method. He also extended the parameter estimation to non-linear storage functions, using a so-called segmented linear curve-line approach which nowadays could be considered as some kind of a linearization of the inherently nonlinear objective function for the storage. The same author then later (Gill, 1984) explained the use of the proper time lag in this Muskingum- method by considering some specific examples. Mohan (1997) applied a genetic algorithm to estimate the parameters in a nonlinear Muskingum model. Further improvements in the method were made by Perumal and Ranga Raju (1998) who related the parameters of the routing equation to the channel and flow characteristics, so that the former could be varied at every routing time level. An optimization approach has been proposed by Das (2004) who estimated parameters for Muskingum models using a Lagrange multiplier formulation to transform the constrained parameter optimization problem into an unconstrained one. However, Geem (2006), who used an unconstrained BFGS-optimization technique for the same purpose, pointed out that the FD-formulation of the constraints by Das (2004) was not consistent with the continuity equation.

Oladghafari *et al.* (2009) determined the routing parameters of the Muskingum model for three flood events (which are also at the focus of the present study) in a reach of the Mehranrood river in northwestern Iran by the classical (graphical) procedure (Chow *et al*., 1989) and compared the results obtained in this way with those acquired by using the full dynamic wave flood routing method. As expected, the latter simulated the observed output hydrographs better than the Muskingum method. A further improvement to the Muskingum method has been made by Perumal *et al.* (2009) who extended the latter to a multi-linear stage-hydrograph routing method in which model parameters can be varied at each routing time step. The method was verified by laboratory experimental data and the field data of the Tiber River in central Italy.The literature review above clearly shows that research on Muskingum flood routing is alive and well and by no means exhausted. That is, the determination of the optimal routing coefficients in the Muskingum model in a real application is not yet satisfactorily solved and is, thus, still open to debate. This is the issue of the present paper, where two new parameter estimation techniques, namely, nonlinear programming (NLP) and multiple linear regression (MLR), will be applied to the routing of three floods which have already been analyzed by Oladghafari *et al.* (2009) by means of a classical Muskingum model. The routing coefficients obtained by these two parameter optimization techniques will eventually be compared with those used by Oladghafari *et al.* (2009) in his traditional Muskingum routing model.

2 STUDY METHODS

2.1 *Kinematic/diffusion wave / Muskingum wave routing method*

One of the most widely used methods for river flood routing is the Muskingum method (Chow et al., 1989). Although this method belongs to the class of hydrological routing or level-pool routing techniques, whereby a stream section is treated like a reservoir and the classical continuity equation

$$dS/dt = I(t) - Q(t) \tag{1}$$

for the change of storage $S(t)$ as the difference between inflow $I(t)$ and outflow $Q(t)$ is applied – i.e. momentum transport described by the Saint Venant equations is not considered, the numerical implementation in the form of the so-called Muskingum-Cunge method (Cunge,1969) has been shown to be close to a diffusion-wave type of routing method, as derived from an approximation of the Saint Venant equation.

Using the concept of a wedge-/prism storage for a stream reach, whereby the total actual storage is written as a weighted average of the prism-storage $S_{prism}=KQ$ and the wedge storage $S_{wedge}=KX(I-Q)$

$$S=K[XQ+(1-X)(I-Q] \tag{2}$$

where K is a reservoir constant, (about equal to the travel-time of a flood wave through the stream reach), and X a weighting factor, both of which a usually determined in an iterative manner from observed input- and output hydrographs, the discrete Muskingum-equations are directly obtained from the time-discretization of Eq. (1) (Chow *et al*., 1984):

$$Q_{j+1} = C_1 I_{j+1} + C_2 I_j + C_3 Q_j \tag{3}$$

where $j = (1,...,m)$ indicates the time step and C_1, C_2 and C_3 are the routing coefficients, which include the two constants K and X , as well as the time step Δt.

In spite of the many implementations of the Muskingum equations in numerous flood routing codes (e.g. TR-20, SWMM, HEC-1, to name a few), the proper determination, i.e. calibration of the three routing coefficients in Eq. (3) above has always been a challenge in real flood routing applications. Apart from the (graphical) trial-and-error approach, automatic parameter estimation based on linear and/or nonlinear optimization methods (e.g. Gill, 1978; Hosseini *et al.,* 2004) have been applied with some success. In this study we use NLP and MLR, both of which will be formulated in the subsequent sections.

2.2 *General formulation of a constrained nonlinear programming problem (NLP)*

The main purpose of nonlinear programming (NLP) is to find the optimum value of a functional variation, while respecting certain constraints (Luenberger, 1984; Hiller and Liberman, 1995; Avriel, 2003). The NLP-problem is generally formulated as:

$$\begin{aligned} &\min f(x) \\ &s.t: h_1(x)=0, h_2(x)=0,..., h_n(x)=0 \\ &x \in \Omega \subset E^n \end{aligned} \tag{4}$$

Where $f(x)$ is the objective function, x the decision variable, and $h_i(x)$ specifies the constraints. It is assumed that f and $h_i(x)$ are continuous functions. It should be noted that, depending on the particular problem, instead of *min f(x),* (convex problem), *max f(x)* (concave problem) may also be searched. Also, likewise to most other classical optimization procedures, which are mostly using some gradient-method to search for the minimum of the objective function, the solution of the NLP-problem (4) will not yield an absolute, but only a relative extremum of $f(x)$ at $x=x_0$, i.e. $f(x) > f(x_0)$ for $x\#x_0$. To overcome this deficiency, global minimization methods, such as, for example, the genetic algorithm, can be used, as it was done by Mohan (1997), mentioned in the introduction. The computation of the optimum (minimum) values for the nonlinear constrained problem (4) is a difficult task, and depends on the form of the objective function and of the constraints. Many general nonlinear problems can be solved, for example, by application of a sequence of Linear Programming (LP) or, in cases that $f(x)$ can be written as a quadratic, by Quadratic Programming (QP)- techniques. A particular difficulty in solving a NLP-problem is due to the presence of the constraints. One way, to overcome this burden is to convert the constrained optimization problem into an equivalent unconstrained optimization problem by setting up a Lagrangian multiplier (penalty)- formulation (e.g. Luenberger, 1984), whereby in each iteration step (stage) of the minimization procedure the total sum of the original objective function $f(x)$ and a penalty-weighted sum of the constraints $h_i(x)$ is now minimized, i.e. Eq. (4) is changed to

$$\min f(x) + \mu(k)\left[E(x(k))\right]^{\rho} \qquad k = 1,2,....,k_{\max} \tag{5}$$

for each stage k, where $x(k)$ is the solution at that stage; μ(k) is the penalty parameter; and $E(x(k))$ is the sum of all constraint-violations to the power of ρ, taken as $\rho=2$ here.

With Eq.(5) the constrained minimization problem (4) in converted into a sequential unconstrained minimization problem that can be solved by classical nonlinear minimization procedures. Here the Hooke and Jeeves (HJ) line search method (Hooke and Jeeves, 1961), as implemented in the WINQSB-software, is used. Unlike most descent-type minimization procedures commonly used, the HJ-method does not require the gradient of the objective function, which for the penalty function (5) is not easy to compute.

Before beginning the iteration procedure in the HJ-method, a stopping tolerance value δ is specified, and a starting solution x(1) is provided to the NLP code. Iterations $k=1,2..k_{max}$ continue until $\mu(k)*E(x(k))^{\rho} < \delta$, then $x(k+1)$ is the optimal solution, otherwise $\mu(k+1) = \beta*\mu(k)$, with β a constant.

2.3 *NLP-formulation of Muskingum flood routing*

The NLP -problem for flood routing is formulated here as follows

$$\min f(x) = \left|V - \hat{V}\right| \tag{6}$$

under the constraints

$$Q_{j+1} = C_1 I_{j+1} + C_2 I_j + C_3 Q_j \qquad , j = 1,2,...,m \tag{7}$$

and

$$V = 0.5\sum_{i=1}^{n}(Q_i + Q_{i+1})\,\Delta t \tag{8}$$

i.e., the goal in Eq. (6) is to minimize the absolute difference between the volume $\hat{V}$ of the observed input hydrograph to the reach and the volume V of the routed output hydrograph. The constraints (7) are nothing else than the discrete formulation of the continuity equation (1) in the form of the Muskingum equations (3) for the input- and output discharges measured at the discrete times $j=1,2,...,m$. In the present application the hydrographs have been sampled at $m=24$ times. Finally Eq. (8) provides the relationship to compute the volume V in each iteration step of the minimization procedure from the discharge rates Q.

2.4 *Multiple linear regression (MLR) method*

In the multiple linear regression (MLR) model, the Muskingum equation (3) is read like a linear regression equation for the dependent (response) output variable Q_{i+1} as a function of the three independent variables I_{i+1}, I_i (measured input hydrograph) and Q_i. (measured output hydrograph). With this the MLR-model can be stated as:

$$Q_{j+1} = C_1 I_{j+1} + C_2 I_j + C_3 Q_j \ + \varepsilon \qquad , j = 1,2,...,m \tag{9}$$

where ε is the remaining, unexplained error term in the model, due to errors in the data as well as in the model formulation itself.

The MLR-model is solved for the regression (routing) coefficients C_1, C_2 and C_3 by classical least squares, i.e. by minimizing the quadratic error ε^2 in Eq. (9). Numerically this can be done either by solving the normal equations derived from Eq. (9) (e.g. Draper and Smith, 1981) or, in a more stable manner, by QR-decomposition (Golub and Van Loan, 1996) of the over-determined system of equations (9). The latter method has been employed using the MATLAB software.

3 STUDY AREA AND FLOOD EVENTS USED

The study area is located along the reach of the Mehranrood River in the Azarbayejan-e-Sharghi province in northwestern Iran between the two hydrometer stations Hervi (upstream) and Lighvan (downstream). The stream distance between these two stations is 12280 meters.

Three flood events that occurred on April 6, 2003, June 9, 2005 and May 4, 2007, respectively, were selected for the flood routing experiments. Input hydrographs for the simulations are the observed discharges at Hervi gage station and the output hydrographs those at station Lighvan.

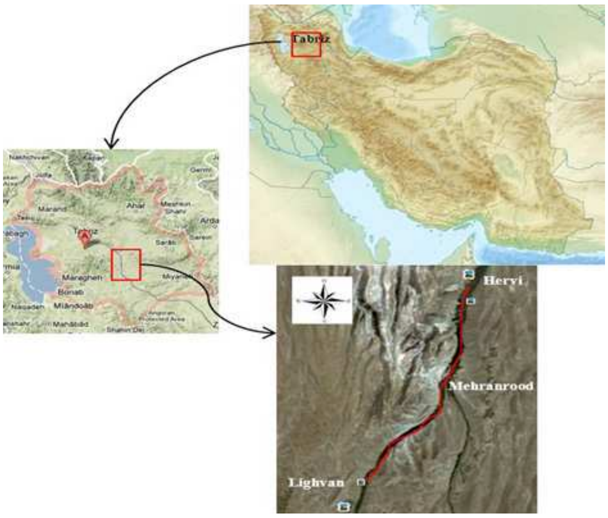

Figure 1. Study area with the Mehranrood stream reach between the gage stations Hervi and Lighvan.

4 RESULTS AND DISCUSSION

4.1 *General set-up of the flood routing computations*

The NLP- and the MLR- flood routing method have been applied to the three flood hydrographs mentioned earlier and the results obtained compared with those of Oladghaffari *et al*. (2009) who used the traditional Muskingum method. Firstly, the optimal calibration of the three routing coefficients C_1, C_2 and C_3 in Eq. (3), i.e. the decision variables in NLP, or the regression coefficients in MLR, has been done with the observed input (at station Hervi) and the routed output (at station Lighvan) hydrographs of the April 6, 2003 flood event. After successful calibration, these routing coefficients have been used in the subsequent verification of the other two flood events, June 9, 2005 and May 4, 2007.

To assess the reliability of the routing methods, the coefficient of determination R^2 and the average absolute error AAE have been used.

4.2 *Optimal calibration of routing coefficients using the April 6, 2003 flood event*

For the NLP-method, the primary parameters used in the penalty function formulation (see Section 2.2) are listed in Table 1, and these have been used in all three applications.

Table 1. Parameters used in the NLP-penalty function method.

Parameter	ρ	δ	β	$\mu(1)$	$X(1)$
Value	2	0.0001	0.1	1	0

Table 2 shows the results of the calibrations of the optimal routing coefficients C_1, C_2 and C_3 with the NLP- and the MLR- method, using the hydrograph data of the April 6, 2003 flood event. In addition, the C-coefficients of Oladghaffari *et al*. (2009), who used a classical graphical procedure (Chow *et al.*,1989) together with some visual fitting of the simulated to the observed output hydrograph for determining the optimal Muskingum parameters K and X and, subsequently, the C-coefficients, are listed. One may notice that, whereas the C_3-coefficients are more or less similar for the three routing methods, larger differences for C_1 and C_2 exist, especially relative to the ones of the Muskingum method.

Table 2. Optimal NLP-, MLR-, and classical Muskingum- routing coefficients for the April 6, 2003 flood event.

Method	C_1	C_2	C_3
NLP	0.0877	0.2886	0.6239
MLR	0.1043	0.2347	0.6612
Muskingum	0.2606	0.0758	0.6636

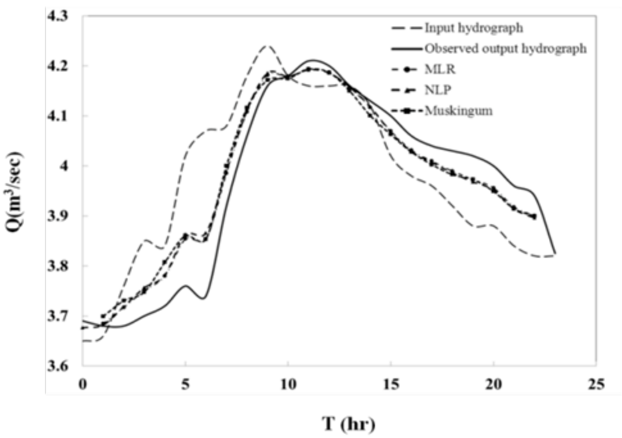

Figure 2. Flood routed hydrographs using NLP-, MLR- and classical Muskingum for the April 6, 2003 flood event.

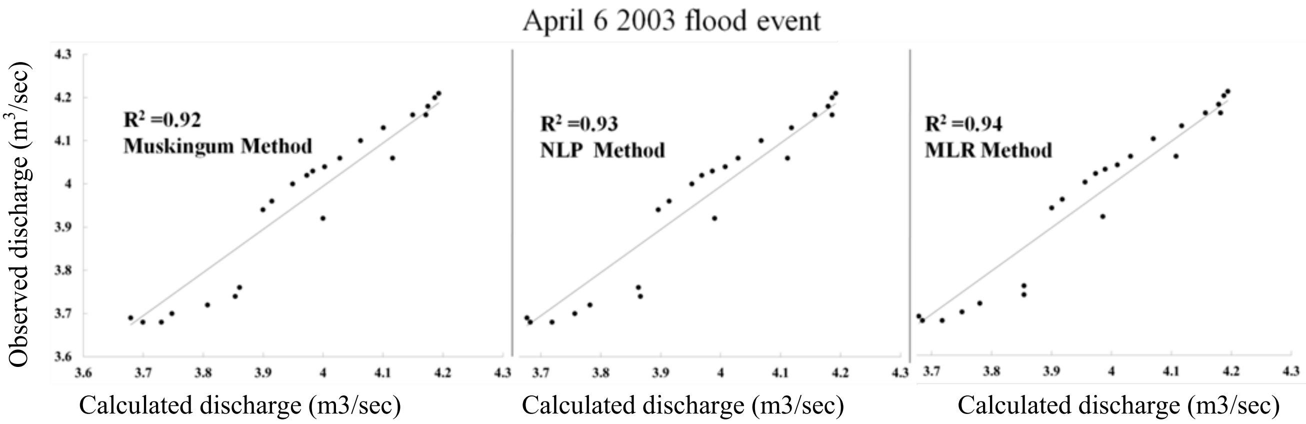

Figure 3. Calculated versus observed discharge for Muskingum, NLP and MLR for the April 6, 2003 flood event.

The corresponding hydrographs for that flood event are shown in Figure 2, where the one obtained by the classical Muskingum method has been extracted from Oladghaffari *et al.* (2009). The observed and calculated discharges at the reach-outlet station Lighvan for the three routing methods are shown, together with the fitted regression lines and the computed value for R^2 (see Eq. 10), in Figure 3.

From the visual inspection of the hydrographs in Figure 2, and despite the noted differences in the optimal routing coefficients (see Table 2), all three routing methods appear to be more or less equally good. However, the regression lines of Figure 3 and the corresponding calculated R^2 clearly indicate the MLR and NLP as slightly superior over traditional Muskingum in fitting the observed hydrograph.

4.3 *Verification of the flood routing methods with the June 9, 2005 flood event*

The first verification of the three routing methods has been done for the June 9, 2005 flood event, with the optimally calibrated routing coefficients of Table 2, i.e. the normal Muskingum method (Eq. 3) has been used with these coefficients to route the input hydrograph to the reach-outlet station Lighvan.

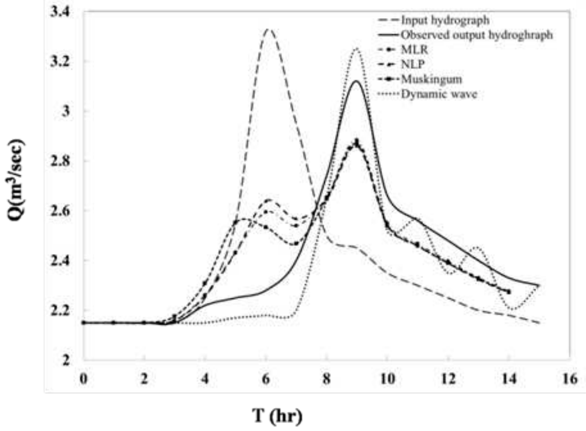

Figure 4. Hydrographs of verification of optimally calibrated routing models for the June 9, 2005 flood event. In addition, the dynamically routed hydrograph of Oladghaffari *et al.* (2009) is shown.

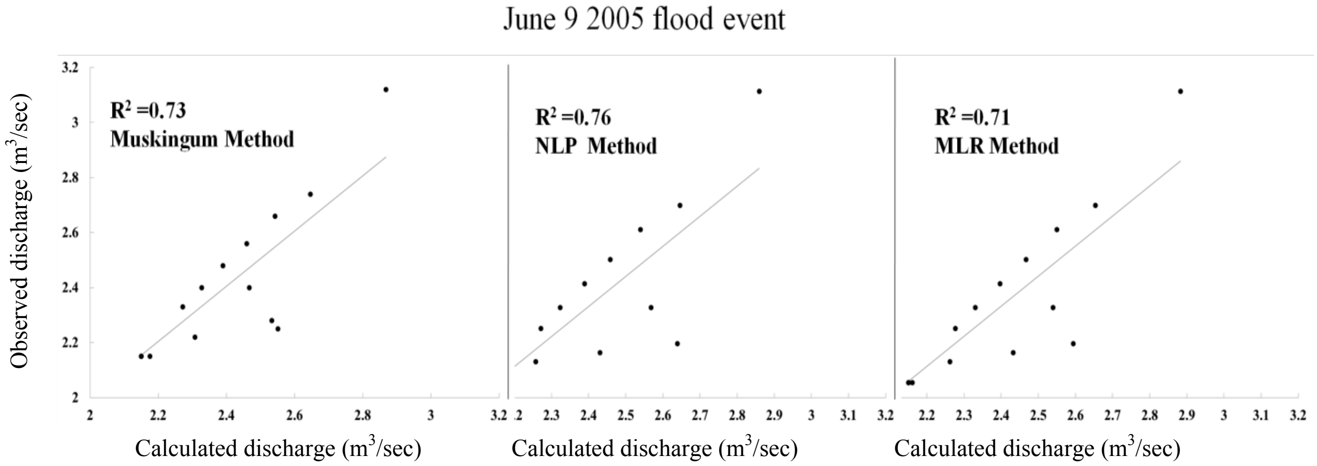

Figure 5. Similar to Figure 3, but using the June 9, 2005 flood event as verification.

The results of this verification run are shown, equivalently to the corresponding figures for the calibration run in the previous section, in Figures 4 (hydrographs) and 5 (regression lines). One can notice from these figures that for this June 9, 2007 verification flood event, somewhat expectedly, the simulated hydrographs for all methods agree less well with the observed ones, than has been the case for the calibration event of the previous section. However, large undulations in the output hydrographs for all three methods are now observed, which may be an indication of solution instabilities in these kinematic-wave type methods - both the NLP- and MLR- approach are actually also based on the Muskingum formulation (see Eqs. 7 and 9) - to route this sharp rising flood wave, which was generated by an intense storm event, as they are frequent in this part of Iran in the early summer.

To investigate this allegation further, we have added in Figure 5 the dynamically routed hydrograph of Oladghaffari *et al.* (2009), i.e. the one computed using the full Saint Venant equations. The wavy form of the rising limb of the simulated hydrograph has now disappeared and the peak of the observed hydrograph is now much better hit than with the three Muskingum-type routing methods.

4.4 *Verification of the flood routing methods with the May 4, 2007 flood event*

The second verification run has been done in a similar manner on the hydrographs of the Mai 4, 2007 flood event. The corresponding results are shown in Figures 6 (hydrographs) and 7 (regression- lines). For this May 4, 2007 verification flood event, with a much wider input hydrograph than that of the June 9, 2005 event, the agreement of the modeled to the observed hydrographs is now again much better than has been the case for the event before. Figure 6 shows that the output hydrographs for the three routing methods are essentially congruent.

The good performance of this flood routing verification is also corroborated by the high R^2- values > 0.9 obtained now (Figure 7), when compared with those of Figure 5, where R^2 was barely > 0.7. The results of Figure 9 indicate also that the differences in R^2 are only minor.

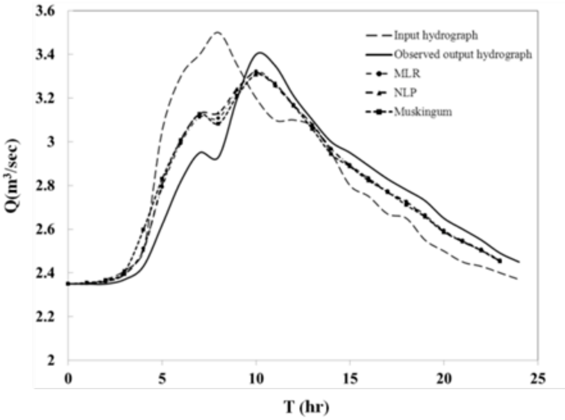

Figure 6. Hydrographs of verification of optimally calibrated routing models for the May 4, 2007 flood event.

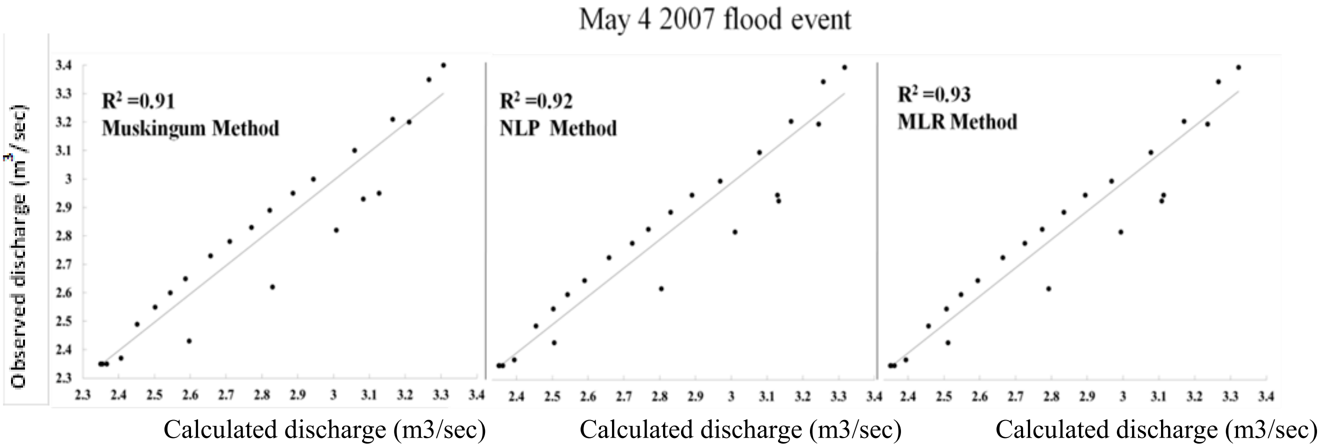

Figure 7. Similar to Figure 5, but using the May 4, 2007 flood event for verification.

5 SUMMARY AND CONCLUSIONS

In Tables 3 and 4 the most salient results of all three flood routing simulation sets (one calibration- and two verification runs) are summarized.

Table 3 lists the observed and calculated peak discharges (taken from the corresponding output hydrographs) as well as the percental errors for the NLP- , MLR- , and Muskingum flood routing methods, from which one can infer all three methods work more or less equally well, with some marginal advantages for the two automatic calibration methods NLP and MLR.

In Table 4 the statistics of the fits of observed discharge hydrographs by the NLP-, MLR- and Muskingum method are listed, namely, the average absolute error AAE (Eq. 11), in addition to the R^2-values, already discussed. Obviously the AAE values inversely reflect the R^2-values, so they do not appear to represent an extra performance indicator.

Based on these results we conclude that the two new parameter optimization methods proposed here for the automatic calibration of the routing coefficients in the widely used Muskingum flood routing method, namely, the nonlinear NLP-technique and the linear MLR-method are powerful and reliable procedures for flood routing in rivers. Although their precision is not necessarily better than that of the traditional Muskingum method – which is of no surprise, as these two methods are also based on the Muskingum formulation - they may be more conveniently used than Muskingum, where suitable routing coefficients (usually the storage parameter *K* and the weighting factor *X*) are often obtained only after some lengthy trial and error process.

Table 3. Observed and calculated peak discharges (m^3/sec) and errors for NLP- , MLR- and Muskingum flood routing.

Flood	April 6, 2003			June 9, 2005			May 4, 2007		
Parameter	NLP	MLR	Muskingum	NLP	MLR	Muskingum	NLP	MLR	Muskingum
R^2	0.93	0.94	0.92	0.71	0.76	0.73	0.92	0.93	0.91
AAE	0.010	0.010	0.011	0.042	0.038	0.041	0.025	0.023	0.027

Table 4. Statistics of the fits of the observed discharge hydrographs by the NLP-, MLR- and Muskingum method.

Flood event	Observed	NLP	NLP-error (%)	MLR	MLR-error (%)	Muskingum	Muskingum-error (%)
April 6, 2003	4.21	4.19	0.47	4.19	0.47	4.19	0.47
June 9, 2005	3.12	2.86	8.33	2.89	7.37	2.87	8.01
May 4, 2007	3.40	3.32	2.35	3.32	2.35	3.31	2.65

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