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# A Sequential Simulation-Optimization Model for Water Allocation from the Multi-Reservoir System in the Karkheh River Basin System, Iran

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**ABSTRACT:** Sequential Streamflow Routing (SSR), with a control on the reliability of meeting the energy demand is a common method for assessing the energy potential in practical hydropower projects' design and operation in Iran. This approach has resulted in the development of a Reliability-Based Simulation (RBS) model which has originally been used for the design of single-reservoir hydropower projects, but has been extended here to multi-purpose multi-reservoir systems for the evaluation of other demands in an integrated system. This modified RBS-model has been applied here to assess water resource development plans in the large-scale Karkheh river basin system in Iran. The system consists of seven dams that serve for hydropower generation, agricultural water supply and other environmental purposes. The first step in the RBS-model consists in the estimation of the production capacity of the power plants and the areas of irrigation systems under development, so that the target reliabilities in meeting the energy- and agricultural demands are achieved. This requires the use of the simulation model in a sequential scheme which iteratively adjusts the design variables of the power plant production capacities as well as of the irrigation areas, in order to arrive at the desired reliability levels. For this purpose, the MODSIM river basin decision support system has been used to simulate the system performance, in which sequential single-period Network Flow Programs (NFP) are employed to determine water amounts allocated to different uses, according to their priorities, for each set of design variables. Furthermore, using the custom coding facility in MODSIM, the firm or reliable hydropower energy *FE* to satisfy the demands has been computed. The results show that the RBS/MODSIM- model is capable of simulating specific operating policies relevant to the hydro-system's objectives and provides the possibility of comparing different scenarios of the Karkheh river basin reservoir system.

*Keywords: Simulation, Optimization, Reliability, Hydropower energy, Network Flow Program, MODSIM*

## 1 INTRODUCTION

The fast growth in population, as well as the ongoing industrialization and urbanization throughout the world have increased the demand for water for irrigation, hydropower generation and for other domestic and industrial uses. Hence, water resources planning and management has become more important in recent years. Operation of systems targets can include meeting the agricultural, domestic, environmental, industrial and irrigation water demands, besides power generation.

Mathematical and numerical modeling, in particular, allows for the detailed consideration of the complex behavior in reservoir systems operation at low computational costs. The mathematical procedures involved in such a task may involve integer or nonlinear programming models whose solutions require increased computer times when compare with the simpler linear programming models. Such models can incorporate specific operating policies then analyze and solve the problem considering the input conditions which are needed for getting particular, desired results. Rule curves (Lucks and Sigvaldason, 1982), heuristic rules such as the space rule (Bower *et al.*, 1962), New York City (NYC) rule (Clark, 1950), hedging rules (Tu *et al.*, 2003), are commonly used techniques for defining operating policies required in these simulation models.

One particular objective of such a simulation model may be the reliability-assessment with regard to the generation of hydropower and/or water for agricultural purposes. For instance, Roman and Allan

(1994) performed a simulation of a hydrothermal power system, in order to assess its service reliability. The authors evaluated different reliability indices from the sampling of the power outages of the power plant and of the network buses and put these in relation to the water inflows and the demand levels by means of a discrete simulation model. Van Hecke *et al.* (1998) and Sankarakrishnana and Billinton (1995) applied a simulation model for the same purpose, however, that considered only the transmission network. To that avail a hourly sequential simulation model was developed, which was calibrated on the sampled outages of the power plant and the transmission elements. Additionally, the authors used variance reduction techniques to obtain a more efficient simulation process. Afzali and Mousavi, (2008) developed and applied a reliability-based simulation- optimization model for a multi-reservoir hydropower system in the Khersan Hydropower system, Iran and showed that, unlike many single reservoir systems, integrated systems are able to produce stable excess electric energy.

A number of computer models for river basin planning and management have been developed by various agencies and used extensively since the mid-1970s. Many early developments in this field used heuristic weight factors to represent priorities of allocation in the optimization procedures, the latter being mostly based on network flow algorithms (NFAs) which are computationally considerably faster than standard simplex solvers employed in linear programming (Ilich, 2008). The HEC- family of models (HEC-3, HEC-5 and HEC-ResSim), CALSIM (Draper *et al.*, 2004) and MODSIM (Labadie, 1995) are good examples of this class of algorithms. In fact, some of these models can both simulate and optimize a system. This is the case for MODSIM (Dai and Labadie 2001; Campbell *et al.*, 2001; Srdjevic *et al.*, 2004) which has been extensively used worldwide.

The present study attempts to develop a simulation/optimization approach for the operation of the system components in a multi-reservoir, multi-purpose Reliability-Based Simulation (RBS) model. Within this model system, the sequential simulation model MODSIM is run to meet the specific reliabilities. Although MODSIM is primarily a simulation model, its network flow optimization capabilities provide an efficient means of assuring an optimal allocation of flows in a river basin, in accordance with specified water rights and other priorities. More specifically, network optimization techniques are specialized algorithms that perform integer-based calculations on linear networks and these are considerably more efficient than real-number computations and matrix operations employed in standard linear programming codes which are usually based on extensions of the revised simplex method. The MODSIM- model which is embedded in a decision support system (DSS) environment is customized here for application to the complex multi-reservoir system in the Karkheh river basin, Iran.

## 2 THE MODSIM DECISION SUPPORT SYSTEM

MODSIM is a generic river basin management model, developed by Dr. Labadie of Colorado State University in the mid-1970s. The model has been widely used for the simulation of river systems worldwide (Dai and Labadie, 2001; Campbell *et al.*, 2001; Srdjevic *et al.*, 2004). MODSIM is designed for developing basin-wide strategies for short-term water management, long-term operational planning, drought contingency planning, water rights analysis and resolving conflicts between urban and rural stakeholders, and to forestall other environmental concerns. The GUI of MODSIM allows users to create and link river basin network objects on the display. Moreover, it connects the model with the several database management components and an efficient network flow optimization (NFO) model. Users can prepare customized code in Visual Basic.NET or C#.NET languages for direct use in MODSIM.

MODSIM basically simulates water allocation policies in a river basin through the sequential solution of the following linear optimization problem over the planning period:

$$\text{Minimize} \quad \sum_{k \in A} c_k q_k \quad (1)$$

Subject to:

$$\sum_{k \in O_i} q_k - \sum_{j \in I_i} q_j = 0; \quad \text{for all nodes } i \in N \quad (2)$$

$$l_k \leq q_k \leq u_k; \quad \text{for all links } k \in A \quad (3)$$

where  $A$  is the set of all arcs or links in the network;  $N$  is the set of all nodes;  $O_i$  is the set of all links originating at node  $i$  (outflow links);  $I_i$  the set of all links terminating at node  $i$  (inflow links);  $q_k$  is the

integer valued flow rate in link  $k$ ;  $c_k$  are cost weighting factors, or water right priorities, per unit flow rate in link  $k$ ;  $l_k$  and  $u_k$  are lower and upper bounds, respectively.

For the cost factor for the artificial carryover storage link cost, the following formula is used:

$$c_i = -(50000 - 10 * OPRP_i) \tag{4}$$

where  $OPRP_i$  is an integer priority ranking number, ranging between 1 and 5000, with lower numbers indicating a higher ranking, i.e. resulting in less negative costs. Notice that minimization of negative costs in Eq. 1 is equivalent to maximizing flows to the higher ranked water uses. Rather than their absolute values, it is the relative order or ranking of the negative costs that determines how MODSIM allocates network flows. Negative costs can be entered to represent benefits, such as from low head hydropower production, since MODSIM solves a minimum cost network flow optimization problem.

Eqs. (1-3) are solved with an efficient Lagrangian relaxation algorithm based on a dual coordinate ascent, called RELAX-IV (Bertsekas and Tseng, 1994), which is up to two orders of magnitude faster than the revised simplex method used in linear programming. An example of completely circulating network is shown in Fig. 1 which illustrates a single reservoir system with an irrigation demand node.

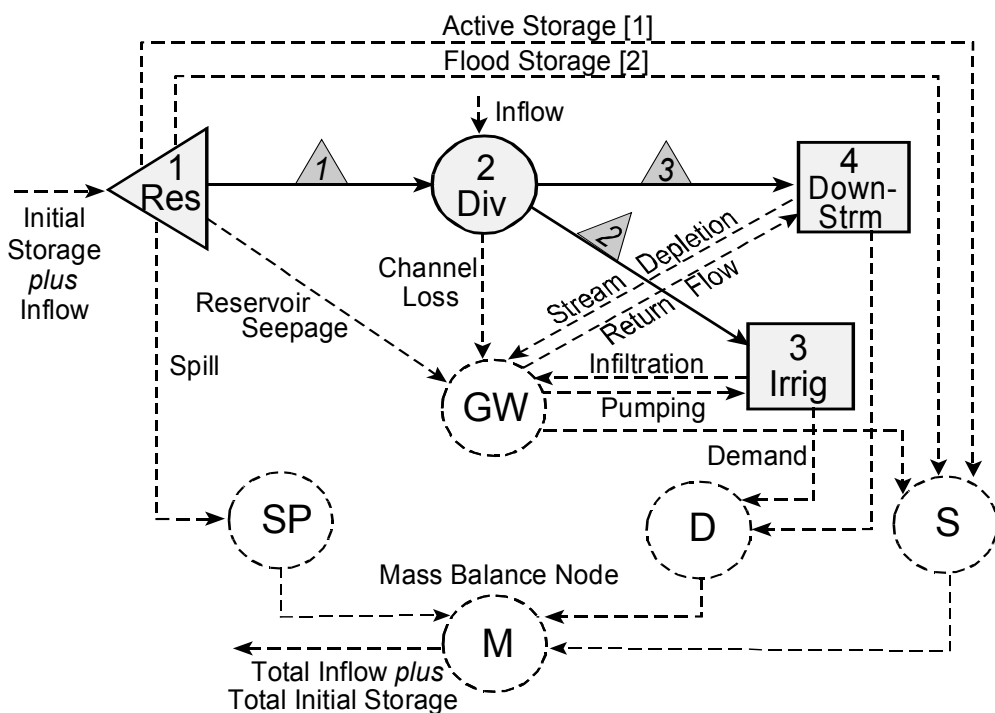


Figure 1. Illustration of MODSIM network structure with accounting nodes and links

### 3 DESCRIPTION OF THE CASE STUDY AREA

The study area is the Karkheh river basin system located in southwest Iran between 33°40'N to 35°00'N latitude and 46°23'E to 49°12'E longitude (upper left inlet of Fig. 2). The Karkheh basin incorporates 7 hydropower systems within 5 reservoirs (Garsha, Kuranbuzan, Sazbonjaryani, Seymareh, Karkhehjaryani) and two multi-purpose reservoirs, Karkheh and Tang-e-Mashoureh. Karkheh and Seymareh damy are constructed and in operation with a fixed installed capacity in their hydropower systems (HPS), whereas the other ones are still under construction or in the planning phase. In the cascade sketch of Fig. 2, D1 denotes the agricultural demand, D2 the domestic and industrial demand and D3 the environmental demand. In fact, downstream of the Karkheh reservoir (KH), there are three major agricultural D1- demand units, KHS, KHT and KHO, using most of the river water, whereas D2- and D3- demands are low, although some minimum environmental flow (D3) must be considered in the allocation process. The KHT demand has separate channel and the water allocated does not pass from the turbine and it does not use to produce the energy. The KHS site receives 10.4 % of the water allocated amount to KHT and KHO as return flow, but its demand is also partly satisfied by underground water.

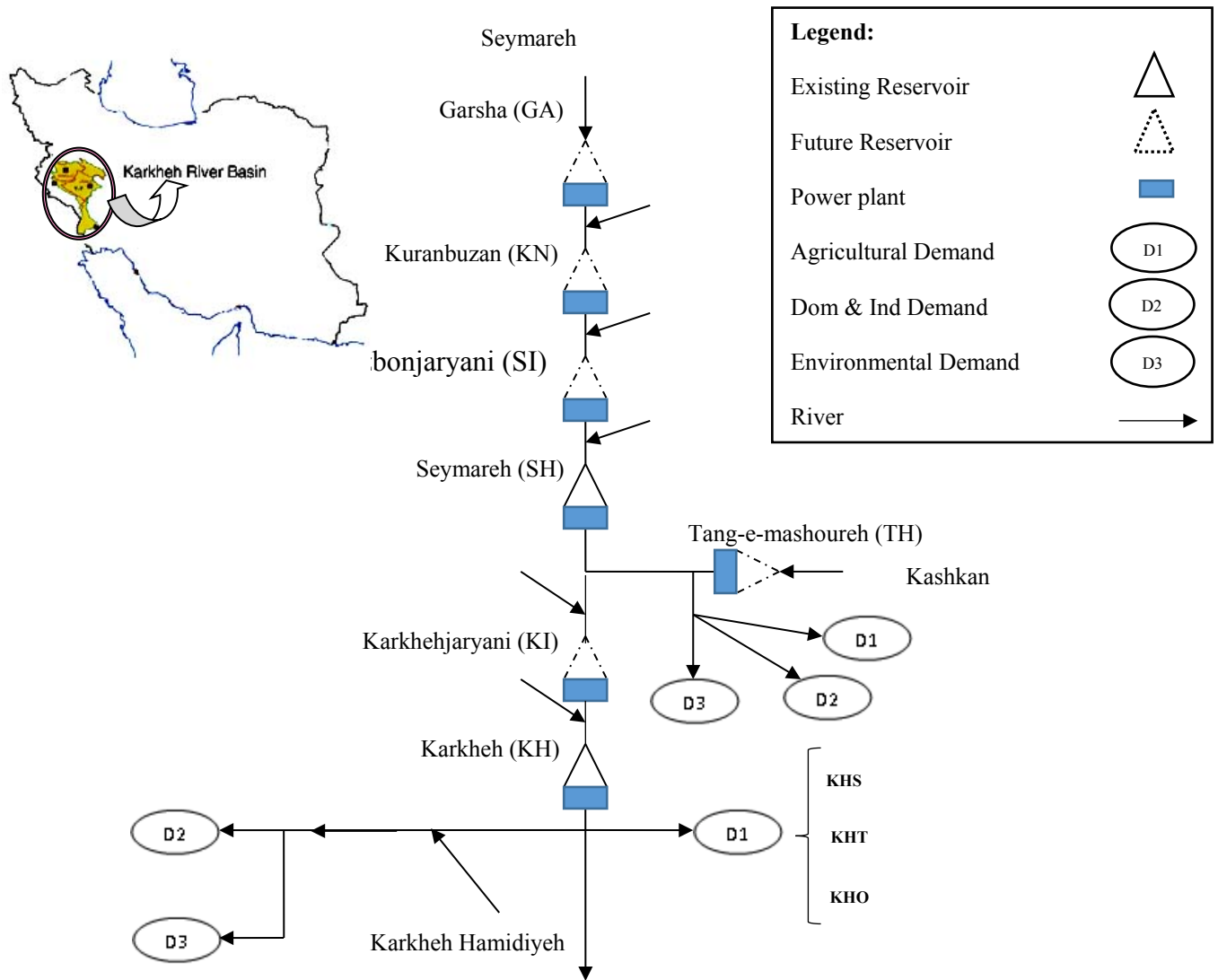


Figure 2. Map and schematic representation of the Karkheh river system with existing and future hydropower units

In the future Tang-e-Mashoureh dam, the amount of water transferred to supply the minimum environmental flows does not contribute to hydropower generation. KHS and Tang-e-Mashoureh agricultural demands can be varied with changes in under-irrigated areas. Besides, minimum environmental flows considered downstream of the Tang-e-Mashoureh (TE) and Karkheh (KE) reservoirs are assumed as non-consumptive demands.

Table 1 shows the main characteristics of seven Karkheh system reservoirs. Also, the under-operation Karkheh and under-construction Seymareh reservoirs have fixed values of installed capacities, as well as minimum and normal water storage volumes.

Table 1. Characteristics of the Karkheh system reservoirs

Characteristics/Reservoir	GH	KN	SI	SH	TH	KI	KH
Efficiency of the plant	0.93	0.93	0.935	0.935	0.93	0.93	0.925
Plant factor	0.25	0.25	0.25		0.25	0.25	
Tailwater level (masl)	1097	945	734	593.7	1064	312	115
Head loss (m)	2.6	2.5	2	5	*	1.92	3
Target reliability	0.9	0.9	0.9		0.9	0.9	
Target vulnerability					0.5		0.5

#### 4 IMPLEMENTATION OF RBS IN THE EXTENDED MODSIM MODEL (RBSM)

The Reliability-Based Simulation (RBS) model aims to determine the maximum system's firm energy yield  $FE$  that can be produced in a hydropower system for a given level of reliability (usually taken as

90%). MODSIM does not have a specific operation policy for monthly hydropower generation, but the former can be defined by custom code in MODSIM. Fig. 3 illustrates MODSIM GUI for a single reservoir hydropower system, wherefore the Hydropower Standard Operation Policy (HSOP) is written in custom code and then named “extended MODSIM”. The hydropower demand should be specified as flow-through demand in MODSIM.

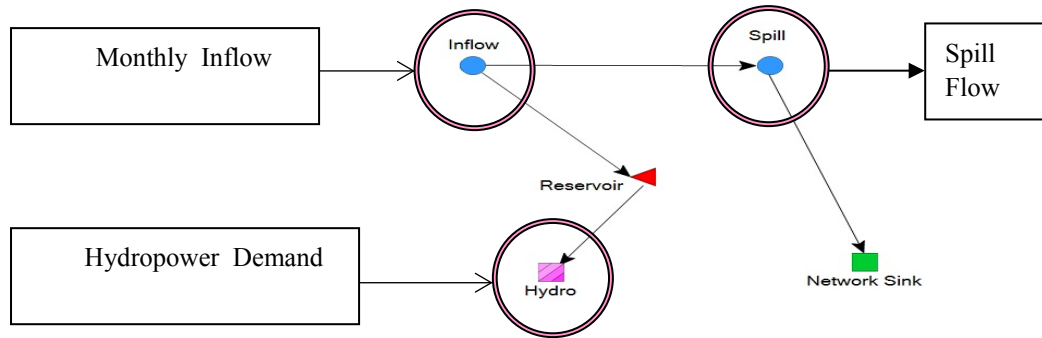


Figure 3. MODSIM GUI for implementation of HSOP in a single reservoir hydropower system

The first step in the modeling procedure is the estimation of the system’s monthly energy yield  $EY(t)$  for month  $t$ , which is computed as

$$EY(t) = I_{cap} \times nHour_{mon} \times PF \quad (5)$$

Where  $I_{cap}$ = power plant’s initial production capacity [MW] ;  $nHour_{mon}$  = number of hours per month;  $PF$ = specified plant factor, defined as  $PF = nHour_{day} / 24$ , with  $nHour_{day}$  the number of hours per day, the power plant is able generate power ( $PF=1$ , for full 24 hour operation).

The energy  $E(t)$  [MWh], produced in each time step (month) is computed as

$$E(t) = 2.725 \times R(t) \times (\bar{h}(t) - h_{Tail} - h_f) \times e_p \quad (6)$$

Where  $\bar{h}(t)$ = average net head on the turbines [m];  $h_{Tail}$ = average tail water level [m];  $h_f$ = total minor and fractional losses in the conveyance structure [m],  $e_p$ = power plant’s efficiency; and  $R(t)$  = turbine release [mcm]. The latter can be computed by simply inverting Eq. (6), assuming that the energy generated in month  $t$ ,  $E(t)$ , must be equal to the assumed energy yield  $EY(t)$ .

Based on the energy generation  $EY(t)$  and the average head  $\bar{h}(t)$  in the reservoir,  $R(t)$  computed from Eq. (6) assigns the amount of water allocated to the Hydro demand node as determined by the inner loop of the NFP- (Network Flow Programing) algorithm in each time step. As  $\bar{h}(t)$ ,  $h_{Tail}$ , and  $h_f$  depend on the turbine release itself, the equation for  $R(t)$  is implicit and must be solved iteratively.

Classical hydrological, i.e. sequential streamflow routing (*SSR*) (Chow *et al.*, 1988), which is based on a discrete version of the flow budget equation, is used to compute the storage in the reservoir during one time step (month). Thus the storage  $S(t+1)$  for a subsequent month  $t+1$  can be written as

$$S(t+1) = S(t) + I(t) - R(t) - Spill(t) \quad (7)$$

Under constraints:

$$S_{min} \leq S(t) \leq S_{max} \quad , R(t) \leq R_{max} \quad , E(t) \leq E_{max} \quad (8)$$

Where  $S(t)$  is the storage for month  $t$ ,  $I(t)$  is the inflow and  $R(t) + Spill(t)$  is the cumulative outflow. The three constraints in Eq. (8) denote (a) upper and lower bounds on the reservoir’s storage  $S(t)$ ; (b) the water release  $R(t)$  from the turbines has an upper bound  $R_{max}$  (defined by the hydraulic characteristics of the turbines), and values higher than that must be released as a spill flow and (c) that the monthly energy production  $E(t)$  cannot exceed the maximally possible energy generation  $E_{max}$ , achieved for a 24-hour plant operation, i.e. obtained from Eq.(5) by setting  $PF=1$ .

Starting with initial values for the named variables, the extended MODSIM solves Eq. (7) for each time step (month) in an iterative manner, to comply with the constraints (Eq. 8).

For the given normal and minimum reservoir operating levels and the specified system power plant factor, the reservoir system operation is simulated and the energy-yield estimated.

In the next step the reliability  $Rel$  of an assumed amount of firm energy  $FE$  is computed.  $Rel$  is defined as the ratio of the number of satisfactory months  $Z$ , i.e. for which  $FE$  can be delivered for at least  $PF * nHour_{mon}$  hours in a month – which, assuming a recommended 4 hours/day- minimum power generation, amounts to  $0.167 * 720 = 120$  hours - to the total operation period  $T$  ( $= 50$  years  $= 600$  months):

$$Rel = \sum_{t=1}^T Z(t) / T \quad (9)$$

A specified target reliability can be reached by changing the power plant's production capacity. This value can be increased or decreased, until the specific reliability is obtained.

The model discussed above can also be applied to a multi-reservoir system. Considering the fact that in a multi-reservoir system, each of the reservoirs can contribute to the system's energy yield, the RBS model can also be expanded for whole reservoir systems by writing Eq. (7) separately for each reservoir.

Also, for a multi-purpose system including some agricultural water demand, a vulnerability parameter  $Vul$  is introduced which is defined as the ratio of the total water deficit = (total water demand  $D(t)$  – water supply  $W(t)$ ) to the total water demand  $D(t)$  during the operation period  $T$ .

$$Vul = \sum_{t=1}^T D(t) - W(t) / \sum_{t=1}^T D(t) \quad (10)$$

Allowable deficits needed for reaching a specific vulnerability parameter can be obtained by adjusting the water demand through an adaptation of the agricultural areas under irrigation.

## 5 RESULTS AND DISCUSSION

Based on historical flows of the Karkheh river observed over the  $T=50$ -year time period 1960-2010 the RBS- model embedded within the MODSIM-model environment has been used to simulate and optimize the cascade of hydropower systems (HPS) for all reservoirs in the Karkheh river basin (see Fig. 2). Here we only report on the modeling results for the planned Garsha reservoir which is the most upstream one of the reservoir system. Entering the simulation/optimization are the observed inflows over the named time period which are shown in Fig. 4 (top panel) and the relevant hydraulic reservoir design parameters taken from planning proposals. The storage changes in the Garsha reservoir simulated in this way are shown in the bottom panels of Fig. 4 and they exhibit, expectedly, an inverse trend as the assumed inflow.

The reliable or firm energy  $FE$  is a function of the assumed reliability (exceedance) ( $Rel$ ) level (Eq.9) and is computed in a here in an inverse way, i.e. for a set of predefined monthly energy yields  $FE = EY(t)$  (Eq.5) the corresponding  $Rel$  are computed which, for the Garsha –HPS, results in the  $EY (Rel)$ - curve of Fig.5. From this curve the firm energy  $FE_{0.9}$ , i.e. the firm energy at the  $Rel = 0.9$  reliability level can be directly retrieved and it turns out be 25.7 GWh for this HPS (see Fig. 5).

Fig. 6 shows the distribution of the total energy for the Garsha HPS over the separate months of a year for the total 50 year simulation time. Because of the strong seasonal variations of the inflows into the reservoirs the total energy is also very variable. Also plotted is the total firm energy available for these months which, per definition, is constant and depends only on the number of days of the month. These barplots indicate that, other than for the wet months, March-May, the amount of excessive or secondary energy, i.e. the difference between total and firm energy, is rather small.

In Table 1 the relevant MODSIM results for all reservoir HPS - two of them acting as multipurpose reservoirs - are listed, where for the latter the vulnerability level  $Vul$  (Eq. 10) for the agricultural irrigation demand is also included. Moreover, with the firm energy  $FE_{0.9}$  computed for a HPS, similar to that for Garsha in Fig. 4, and then using Eq. (5), assuming the specified plant factor  $PF=1/6=0.167$ , i.e. a minimum of 4 hours of operation per day of the HPS the power plant's production capacity  $Icap$ , needed to supply this energy, has also been calculated and is also listed. In particular, Table 1 indicates that the two existing reservoirs, Seymareh and Karkheh, are able to deliver firm energy  $FE$  only at a much lower than the pre-assigned  $Rel = 0.9$  reliability level.

In conclusion, the results show that the MODSIM- simulation/optimization model, embedded in the Reliability-Based Simulation (RBS) -environment is capable of simulating specific operating policies relevant to the hydro-system's objectives, i.e. satisfying a manifold of demands based on ranked priorities namely, for electricity production and agricultural irrigation, and that the model provides the possibility of comparing different scenarios of the Karkheh river basin multi-purpose reservoir system.

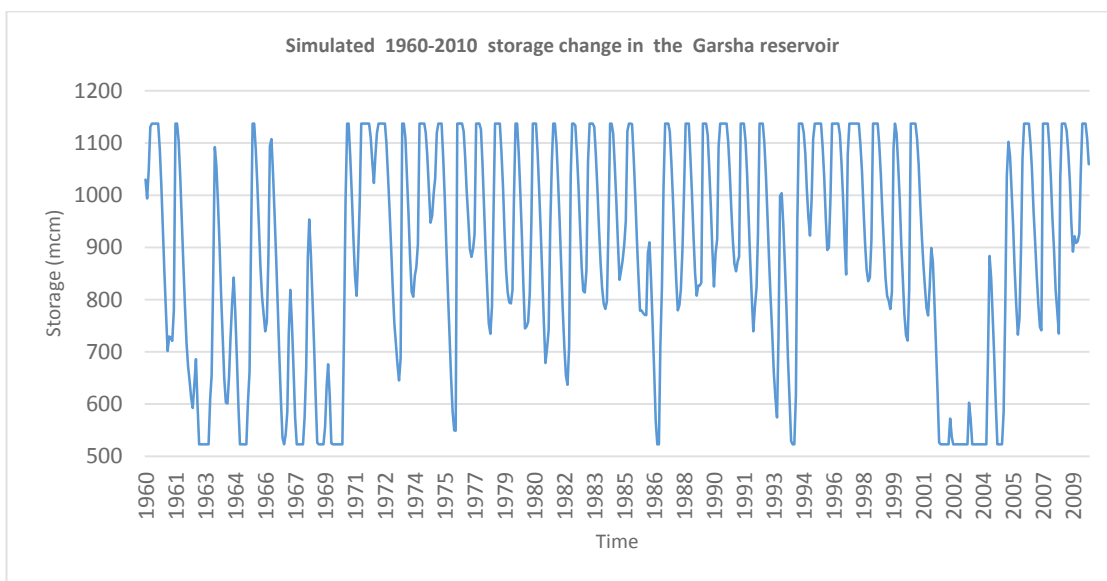
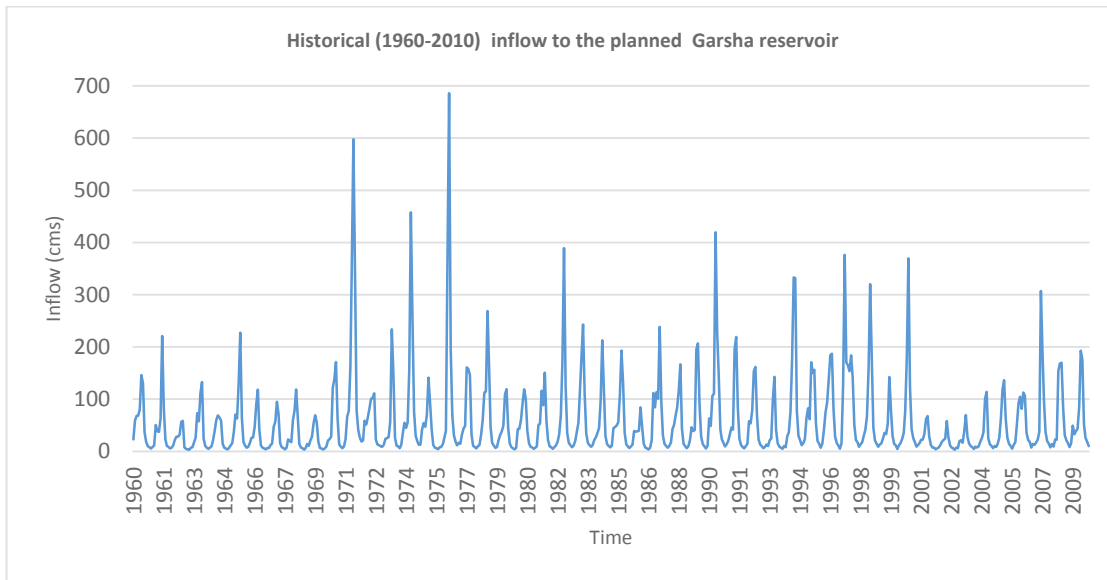


Figure 4. Historical observed inflow (top) and simulated water storage (bottom) in the Garsha reservoir.

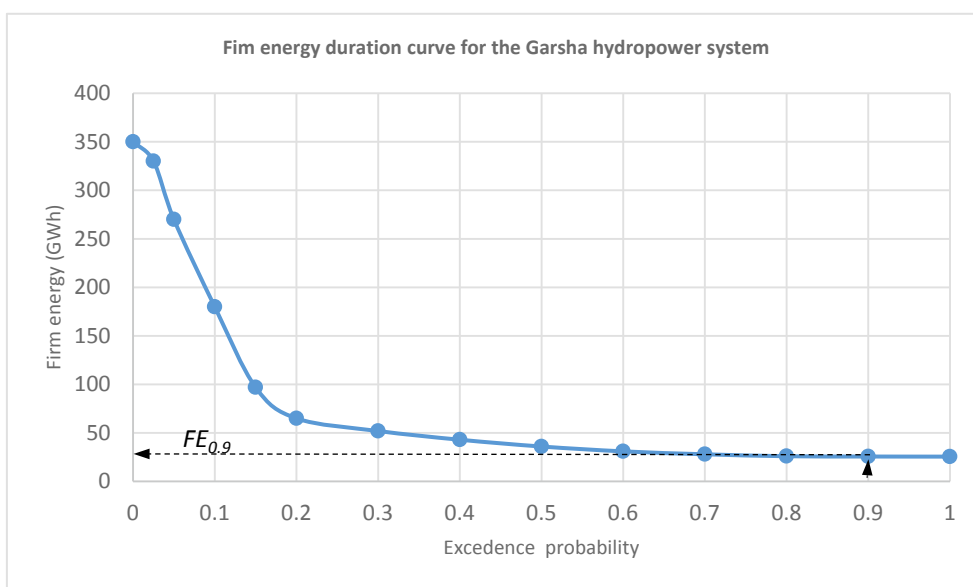


Figure 5. Firm monthly energy generated by the Garsha hydropower system over a  $T=50$ - year operation as a function of exceedance probability (reliability). Target firm energy  $FE_{0.9}$  is the energy obtained for  $Rel=0.9$  and is equal to 25.7 GWh.



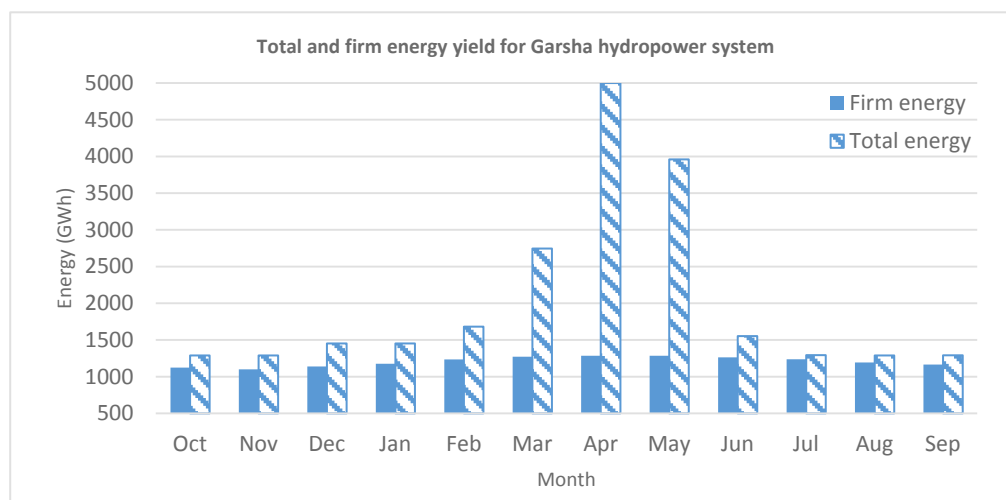


Figure 6. Monthly distribution of Garsha's HPS total and firm energy yield over the  $T=50$ - year operation period.

Table 2. Summary of MODSIM- results for all reservoirs (e =existing) of the Karkheh river basin system.

Reservoir	Type	Vulnerability	Reliability	Irrig. area (ha)	Icap (MW)
Garsha	Hydropower		0.9		214.2
Kuranbuzan	Hydropower		0.9		210.7
Sazbonjaryani	Hydropower		0.9		290.4
Seymareh <sup>e</sup>	Hydropower		0.53		480
Tang-e-mashoureh	Multipurpose	5.01	0.93	25400	51.5
Karkhehjaryani	Hydropower		0.9		254.2
Karkheh <sup>e</sup>	Multipurpose	5.02	0.61	236988	400

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