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Advanced Wave Generation Systems for Numerical Modelling of Coastal Structures

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Abstract: Accurate generation of wave climates in the context of numerical models (and in particular CFD models) is a challenging problem, as these are increasingly used to provide design support to coastal engineering projects. In this paper we briefly present a technique that addresses the generation (and active absorption) of non-repeating wave sequences for modelling storm events in a meaningful manner. This technique includes a spectral window preprocessing method that is used to reduce the computational costs associated with wave generation algorithms. These can be particularly cumbersome for generating storm events using a direct reconstruction of components. Dimakopoulos et al. (2019) demonstrated that numerical cost can be reduced by about 40 times by using $O(10^{1})$ frequencies for wave reconstruction, rather than $O(10^{2}-10^{3})$ which would be required by direct reconstruction to accurately reproduce long wave series, without any noticeable difference in terms of the generated wave signal. The technique is already in use within the context of the computational toolkit Proteus (https://github.com/erdc/proteus) and is it is combined with the CFD modelling module of the toolkit. In this work we present the application of using the aforementioned reconstruction technique, for the following case studies i) wave overtopping over a simple slope and ii) a sliding breakwater case. Results are compared with simulations with repeating sequences to demonstrate the differences and possible ways for further reducing computational cost are discussed.

Keywords: CFD modelling, wave generation, random sea states, numerical wave tanks, shallow water equation, overtopping

1 Introduction

Numerical wave tanks in models have been an established method for supporting the design process of coastal developments and coastal structures. Example layouts for numerical wave tanks for Computational Fluid Dynamics (CFD) models are presented in Jacobsen et al. (2012), Higuera et al. (2013), Chen et al. (2016), and Dimakopoulos et al. (2016). These are using different techniques for generating and absorbing waves such as moving paddles, radiation boundary conditions, and relaxation zones.

In numerical tanks using the relaxation zone method presents significant advantages as i) it is capable of absorbing waves more efficiently than the other methods ii) it does not produce spurious oscillations at the boundary and iii) it is relatively straightforward to implement numerically. The relaxation zone method nevertheless requires the extension of the domain at both the offshore and landward boundaries, to accommodate active and passive absorption zones, respectively. As a result, the increased length of the numerical tank is associated with increased computational cost. Whilst Dimakopoulos et al. (2016) have addressed the increase of computational cost at the passive absorption zone, by reducing mesh refinement in a controlled manner, the cost associated with wave
generation of long random wave sequences was not addressed in that work. This cost is due to the use of multiple frequency components, typically $O(10^2)-O(10^3)$, for generating non-repeating sequences of random waves to simulate the duration of a storm event. Dimakopoulos et al. (2016) used faster trigonometric functions approximations to reduce this cost, but also demonstrated that even with these improvements, reconstructing random wave time-series with the relaxation zone method still takes a significant portion of the total computational time.

In Dimakopoulos et al. (2019), a methodology was proposed to significantly reduce the required number of frequency components for generation of long, random non-repeating wave sequences, using pre-processing techniques with spectral windows, thus reducing the computational times. The methodology is introduced in the Proteus toolkit (http://proteustoolkit.org), successfully used for simulating wave structure interaction problems (de Lataillade et al. 2017, Cozzuto et al. 2018). In this work, we implement the methodology presented in Dimakopoulos et al. (2019) and demonstrate its applications for generating non-repeating wave sequences in two case studies: i) overtopping over a constant slope from the CLASH database (EuroTop 2018) and ii) a sliding breakwater (the latter was presented in Cozzuto et al. 2018). It is also demonstrated that using significantly more reconstruction frequencies does not incur additional computational cost, as was shown in Dimakopoulos et al. (2016).

The remainder of the paper is structured as follows: In Section 2, a summary of the methodology in Dimakopoulos et al. (2019) and implementation within the Proteus toolkit is presented. In Section 3 the aforementioned case studies and associated results are shown, and the effect of using less number of frequency components is discussed. The quality of results are also discussed in terms of comparison with experimental data. Section 5 summarizes the work and discusses potential issues and future work regarding the methodology and the numerical modelling approach.

## 2 Methodology

### 2.1 Linear reconstruction of random wave sequences

Free surface elevation and particle velocity for plane random waves is calculated using linear reconstruction of components according to 1 to 3.

$$\eta(x, t) = \sum_{i=1}^{N} a_i \cos(k_i \cdot \mathbf{x} - \omega_i t + \phi_i)$$ (1)

$$U_h(x, z, t) = \sum_{i=1}^{N} u_{h(i)} \cos(k_i \cdot \mathbf{x} - \omega_i t + \phi_i) \cosh(k_i(d + z))$$ (2)

$$U_v(x, z, t) = \sum_{i=1}^{N} u_{v(i)} \sin(k_i \cdot \mathbf{x} - \omega_i t + \phi_i) \sinh(k_i(d + z))$$ (3)

where $\eta$ = free-surface elevation, $\mathbf{x}$= the position vector at the wave propagation plane, $z$ = the vertical coordinate (vertical axis aligned with gravity), $t$ = time variable, $U_h$ and $U_v$ = the horizontal and vertical velocity, respectively, $a_i$, $k_i$, $\omega_i$, $\phi_i$, $u_{h(i)}$ and $u_{v(i)}$ = the wave amplitude, wave number vector, angular frequency, phase and horizontal and vertical velocity amplitude, respectively, for the $i$-th wave component and $N$ = the number of frequency components. The amplitude for free-surface elevation and velocities are calculated according to spectral distributions for random sea states such as the JONSWAP or Pierson-Moskowitch.

The number of frequency components $N$ needed to ensure a non-repeating time series for a time interval $T$ step of the spectrum in the frequency domain ($\Delta f$) must be set according to equations 5 (Dean and Dalrymple, 1994).

$$T_{\text{max}} = \frac{1}{\Delta f} = \frac{N}{f_m - f_M}$$ (4)
where $\Delta f$ = the frequency sampling interval, and $f_M$, $f_m$ = the maximum and minimum sampling frequency, respectively. Frequencies $f_M$ and $f_m$ can be defined by a spectral band factor $b_f$ so that $f_M = b_f/T_p$ and $f_m = 1/T_p b_f$, where $T_p$ is the peak period. Note that typically $b_f = 2-3$. Taking into account the above, the number of waves in of the non-repeating part of the sequence can be calculated as follows:

$$N_M = \frac{T_{\max}}{T_{\text{mean}}} = \frac{1.1N}{\frac{1}{b_f} - \frac{1}{b_f}}$$

(5)

where $T_{\text{mean}} = \text{mean wave period} \cong 1.1T_p$. Table 1 shows the number of waves in the non-repeating part of a random wave sequence against $N$ for $b_f = 2-3$.

<table>
<thead>
<tr>
<th>Number of linear components</th>
<th>Number of non-repeating waves</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$b_f = 2$</td>
</tr>
<tr>
<td>50</td>
<td>37</td>
</tr>
<tr>
<td>100</td>
<td>73</td>
</tr>
<tr>
<td>500</td>
<td>367</td>
</tr>
<tr>
<td>1000</td>
<td>733</td>
</tr>
<tr>
<td>1500</td>
<td>1100</td>
</tr>
<tr>
<td>2000</td>
<td>1467</td>
</tr>
<tr>
<td>2500</td>
<td>1833</td>
</tr>
</tbody>
</table>

It is observed that a direct linear reconstruction would require at least 1500 or 2000 frequency components ($b_f = 2$ or $b_f = 3$, respectively) to ensure a non-repeating sequence for a storm duration (typically > 1000 waves). This can be particularly time consuming, when using a direct reconstruction of linear components, as was demonstrated in Dimakopoulos et al. (2016).

### 2.2 Reconstruction with spectral windows

Reconstructing the spectrum with spectral windows contributes in a significant reduction of the computational costs, as shown by Dimakopoulos et al. (2019). The method is based on the concept of using a novel technique of spectral windows processing for the linear reconstruction, rather than directly generating the time series of free surface elevation using Equations 1 to 3. Spectral windows processing is typically used for post-processing signals and calculating smoothed spectral distributions (e.g. Welch 1967, Goda 2010). To the author’s knowledge it is the first time it is used for pre-processing a free surface elevation time series and generating the wave field with reduced number of frequencies.

The concept is based on trading calculation operations for storage in memory. The implementation of the method is shown in the following steps:

1. The input wave signals (including free-surface elevation and velocities) is reconstructed once using direct method and appropriate number of frequency components (e.g. > 1500 components for a storm) and before the numerical solution commences.
2. Each wave signal is divided into a finite number of windows containing a reduced number of waves (typically O(10)).
3. Fast Fourier Transform is applied to each window to calculate the spectral energy distribution and the phasing over a finite number of frequency components, which is much less that one required for direct decomposition (e.g. O(10) instead of O(1000)).
4. The spectral properties of each window (spectral energy and phasing) are stored to be available during the numerical solution.
5. During solution, the wave signals (free-surface elevation and velocities) are reconstructed using Equations 1 to 3 but for each window interval rather than the entire time series.
6. A tracking algorithm ensures smooth handover over adjacent windows as the simulation time progresses.

In order to maintain a smooth evolution of the time series, a cosine taper filter is applied at each window, and subsequently adjacent windows are overlapped and the handover occurs at the middle of the overlap region. A graphical representation of the handover between windows is given in Figure 1, more details about the technique is given in Dimakopoulos et al. (2019).

![Figure 1](image_url)

**Fig. 1.** Example of reconstructing a random wave signal with spectral window method. Red dashed and blue solid lines correspond to consecutive windows. The two windows are overlapped in the grey area. Filtered areas are inside the overlap area and outside the vertical dash-dot lines. Handover point is shown with a vertical solid line.

### 2.3 Numerical wave tank

The wave generation module was coupled with the Proteus CFD Toolkit, open-source software which solves transport equations using the finite element method (FEM). The software is freely available at [https://github.com/erdc/proteus](https://github.com/erdc/proteus) through the MIT open-source license.

For the case studies presented herein, the two-phase Navier stokes equations were numerically solved using a coupled level-set / volume of fluid method (Kees et al. 2008, 2009 and 2011). Wave generation and absorption were achieved by using the relaxation zone method. A typical layout of the numerical wave tank with the relaxation zones is shown in Figure 3. The method has been widely employed in a variety of numerical tools such as CFD models (Jacobsen et al. 2012) and depth-integrated models, e.g. BOUSS2D (Nwogu and Demirbilek 2001). The numerical implementation of the relaxation zone made use of the toolkit’s capability to introduce source terms in the Navier-Stokes equations, following the formulation below:

\[
\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = \frac{1}{\rho} (\nabla p + \nabla \tau) + \mathbf{g} + a_\omega \sigma (\mathbf{u} - \mathbf{u}_t) \tag{6}
\]

where \(\mathbf{u}\) and \(\mathbf{u}_t\) = the field and boundary velocity vectors, respectively, \(p\) = pressure, \(\tau\) = shear stress, \(\mathbf{g}\) = gravity vector, \(\rho\) = density, \(a_\omega\) = penalty term coefficient, \(\sigma\) = the relaxation function (varying from 1 at the external boundaries to 0 at the internal boundaries of the relaxation zone – see Figure 1), \(\omega\) = the wave angular frequency and \(\nabla=\left(\frac{\partial}{\partial x},\frac{\partial}{\partial y},\frac{\partial}{\partial z}\right)\), with \(x, y, z\) = the spatial coordinates. Following the dimensional analysis performed in Dimakopoulos et al. (2019), \(a_\omega\) can be set equal to about 5. The relaxation zone function has the exponential form proposed by Jacobsen et al. (2012).
The introduction of these terms in the Navier stokes equations allow for the active wave generation and absorption of outgoing waves at the boundaries. This technique was benchmarked for regular waves against radiation boundary conditions (Higuera et al. 2013) and a different version of the relaxation zone (Jacobsen 2012; Dimakopoulos 2016). Results in Figure 2 demonstrate that the method is more efficient that the radiation boundary conditions. Results from Dimakopoulos (2016) are better, and this is because in the latter, the relaxation zone is applied for the mass conservation equation. This has been shown to increase absorption efficiency (Nwogu and Demirbilek 2001). Absorption efficiency for random wave simulations is 7%, according to Dimakopoulos et al. (2019) for waves with 3% steepness at peak frequency.

Reconstruction with spectral windows can be applied for the calculation of the time series in each cell the generation zone, provided that the overlap and filtered areas are chosen appropriately so as to ensure spatial coherency of the time series (free-surface elevation, velocities) throughout the generation zone. In Dimakopoulos (2019), it was recommended that the filtered and the overlap area take 10% and 70% of the total duration of the window, respectively.
3 Case studies

3.1 Wave overtopping

A 2D numerical tank is set-up to carry out the simulations of wave propagation and overtopping. The geometry of the tank is presented in Figure 4, along with relevant horizontal and vertical dimensions. The main components of the geometry are the generation and absorption zones, the wave propagation zone before the obstacle, the collection tank and a drainage pipe to ensure that the MWL in the landward tank is not decreasing due to the volume of water that overtops. The obstacle can be described as a constant-slope, positive freeboard impermeable dike with crest height $R_c=0.1$m and slope $\tan\theta=1/4$. The leeward side of the obstacle is designed as a vertical wall, with zero crest width (sharp-crested structure). Note that crest width has no influence on overtopping discharge, unless the width is excessively large (EuroTop 2018). The mean water level is equal to $\text{MWL}=0.4$m, with the coordinate system being at the bottom left corner of the domain. Random waves with $H_s=0.17$m and $T_p=2.3$s were generated for a sequence of 500 waves, using refinement of 100 mesh elements per wavelength (typical triangle size is 0.045 m). This case study corresponds to the geometry of one of the tests encountered in the CLASH database (EuroTop 2018) so there are experimental data available for comparison.

Two cases were simulated by using $N=50$, and $N=2000$ linear components for the window reconstruction. For each of the cases, a number of 500 waves was simulated. Snapshots showing important overtopping events for $N=2000$ are shown in Figure 5. Figure 6 shows the free-surface elevation at the exit of the generation zone and Figure 7 shows the corresponding wave spectrum. Direct comparison between the time series is not possible, as the non-repeating interval time-series is different and this makes it impossible for the $N=50$ case to have exactly the same wave sequence with $N=2000$. Comparison of the wave spectra nevertheless shows that the target spectrum is considerably better matched by using $N=2000$. Note that the peak of the theoretical spectrum is not exactly captured by the case with $N=2000$, as there is energy shifted towards higher and lower frequencies than the peak, due to nonlinear effects.
The overtopping discharge was calculated at the tip of the slope by integrating the instantaneous net water flux. In Figure 8 and Figure 9 the instantaneous and accumulative discharges are presented, respectively. From the instantaneous discharges, it is evident that the overtopping signal for N=50 is roughly repeating itself every ~100 seconds whilst the case N=2000 exhibits a non-repeating behaviour throughout the simulation. It is important to stress that the computational time required for both simulations to finish is practically the same within ~2% margin (~1.6 days in 14 processors), using exactly the same mesh refinement. This further demonstrates that calculating the random wave signal with N=2000 rather than N=50 does not increase the computational time. This is a significant improvement over the direct method, which, according to Dimakopoulos et al. 2016, causes a ~40% increase of the total computational time when increasing N from 50 to 500.
The mean overtopping discharge is about the same for both cases (~4 l/m/s), with the simulation that used N=2000 frequency components being slightly larger (q=4.0 l/s) than N=50 (q=3.9 l/s). The measured discharge for this case is 6 l/m/s. Given that:

   i) these numerical tests were run as “blind” benchmarks (the CLASH data base contains minimal information for the experimental), and
   ii) mean wave discharge in the context of designing structures is meaningful as a typical order of magnitude rather than an “exact” value,

then results can be considered satisfactory. Further improvement may be achieved by e.g. refining the mesh or retrieving the exact conditions of the experiment.

The distribution of the individual volumes of the overtopping events, in descending order are shown in Figure 10. It is observed that using N=2000 frequency components, results in a greater number of extreme events in terms of the individual overtopping volume produced. Overtopping is an irregular process and that the mean overtopping discharge is probably not capable of fully describing it (Eurotop 2018). The volume of individual overtopping events can describe how intense or even dangerous overtopping can be, with respect to the specific wave conditions applied and the structure’s geometry, so it is important to provide reliable predictions of these within the context of using numerical models for coastal engineering design.
3.2 Caisson breakwater - sliding

This case was originally simulated in Cozzuto et al. (2018). The set-up of the numerical tank is shown in Figure 11. The case was using random waves \( H_s = 0.13 \text{ m}, \ T_p = 1.3 \text{ s} \) and it is expected to generate a significant sliding event (~1 mm sliding or more) over 15 waves on average. In this paper, the simulation is repeated, using 20 frequency components rather than 2000, which was the case in Cozzuto et al. (2019). Horizontal caisson displacements over time are shown in Figure 12 and it is evident that the vibrating component is dominant. Significant sliding events (~1 mm or more) do not occur at an interval of 45 s (~35 waves).

![Fig. 11. Layout of numerical flume for sliding caisson tests. Units in m.](image)
4 Discussion and future work

Regarding the overtopping tests performed herein, it is observed that using 2000 frequencies for the reconstructions ensures a non-repeating sequence of overtopping events, whilst using 50 frequencies results in a repeating sequence at about 100 s. The proposed technique ensures that the use of 2000 frequencies within the context of the relaxation zone method remains cost-efficient. This is an improvement of the state of the art, as sampling a with spectrum 2000 frequencies and using a direct reconstruction method to produce the signal results to an unreasonable increase of computational time.

The merits of using non-repeating signals are clearly demonstrated in this work. Generating waves with a non-repeating time series for typical storm durations results in predicting significantly better spectral characteristics, and in capturing higher individual volumes over the duration of the test. Results of mean overtopping discharges were compared with the CLASH database and whilst these were in the right order of magnitude, further tests are recommended to address mesh sensitivity and wave calibration, for obtaining more precise results. For the sliding breakwater case, it is also demonstrated that not using statistically meaningful time series may result in underestimating the sliding displacements.

This paper addressed the issue of time consuming wave generation methods for modelling long wave sequences by proposing a new technique relying on pre-processing the time series with spectral windows. This technique is found to be practical for simulating case studies in 2D numerical flumes, as test results from a full storm duration can be obtained in few days. With respect to simulating 3D problems, it is anticipated that the simulations are still time-consuming, due to the sheer duration of the simulation itself (not because of the wave generation method). It is believed that, in this case, there is scope for research on alternative wave theories that would produce representative results for wave overtopping, without having to use long simulations, such as by applying the NewWave theory (Tromans et al. 1991). Similar issues are encountered when quantifying wave loads and sliding displacements and further work should be performed to explore the possibility of conducting shorter simulations. This said, approaches including 2D simulations or reduced length of wave series could be applied in early design stages and as a measure to reduce time and cost for optimizing the structure layout in early stages. Before or during construction, physical modelling or high resolution CFD modelling are recommended to evaluate performance of the structure under full storm conditions.
References


