
Conference Paper, Published Version

Folke, Frederik; Haehne, Angela; Kopmann, Rebekka

Numerical modelling of scale effects

Verfügbar unter/Available at: https://hdl.handle.net/20.500.11970/105190

Vorgeschlagene Zitierweise/Suggested citation:
Folke, Frederik; Haehne, Angela; Kopmann, Rebekka (2018): Numerical modelling of scale effects. In: Bacon, John; Dye, Stephen; Beraud, Claire (Hg.): Proceedings of the XXVth TELEMAC-MASCARET User Conference, 9th to 11th October 2018, Norwich. Norwich: Centre for Environment, Fisheries and Aquaculture Science. S. 15-22.

Numerical modelling of scale effects

Frederik Folke, Angela Haehne and Rebekka Kopmann
Department of Hydraulic Engineering
Federal Waterways Engineering and Research Institute
76187 Karlsruhe, Germany
frederik.folke@baw.de

Abstract— **There is a long tradition of using hydraulic and morphodynamic models for river engineering purposes. Beside numerical models laboratory scale models play a crucial role, both to gain deeper insight into physical mechanisms and to study complex system behaviour and site-specific issues.**

In terms of river engineering all laboratory models are scaled in space and, thus, in time. One generally accepted method in scale models is to use different scaling factors in length and height resulting in exaggerated river geometry to ensure the similitudes of the main physical processes and to consider limitations of laboratory space. The exaggeration factor ranges typically from 1 to 2.5 for hydraulic laboratory models at the BAW, but 8 times larger factors can be found in literature.

In the present paper the scale effects, in particular the exaggeration, of a long fluvial model (Froude scaling) of the Middle Rhine with an exaggeration of n=1.2 are estimated numerically. A three-dimensional hydrodynamic-numerical (3D-HN) model of the laboratory model is set up and calibrated to the laboratory model measurements. The 3D-HN model is resized to nature-scale and to laboratory scale without exaggeration. The numerical results are analysed. Beside the global water level the secondary currents are of particular interest. The scale effects are estimated both qualitatively and quantitatively. The study has shown that the light exaggeration of the laboratory scales improves the results, but the improvement is small compared to the errors due to the scaling from nature-scale to laboratory scale.

I. INTRODUCTION

There is a long tradition of using hydraulic and morphodynamic models for river engineering purposes. Beside numerical models laboratory scale models play a crucial role, both to gain deeper insight into physical mechanisms and to study complex system behaviours and site-specific issues. In terms of river engineering all laboratory models are scaled in space and, thus, in time. This scaling results in so-called scaling effects. Additionally the force ratios have to be scaled using dimensionless numbers, such as the Reynolds number (ratio of inertial forces to viscous forces) or the Froude number (ratio of inertial and gravitational forces). It has to be taken into account that only two similitudes, e.g. the geometrical and the Reynolds number or the Froude number, can be fulfilled in a laboratory scale model using the same fluid and the same gravitational acceleration. For the other force ratios it has to be checked, that both systems are in the same regime, e.g. subcritical or fully turbulent. Other effects such as the surface tension (Weber number) can typically be neglected.

One generally accepted method in scale models is to use different scaling factors in length and height resulting in exaggerated river geometry to ensure the similitudes of the main physical processes and to consider limitations of laboratory space. This is quantified by the ratio, n, of the vertical length scaling factor, M_{Lv}, to the horizontal length scaling factor, M_{Lh},

$$n = \frac{M_{Lv}}{M_{Lh}}. \tag{1}$$

This ratio ranges typically from 1 to 2.5 for hydraulic laboratory models at the BAW. In literature, higher values can be found, such as 20, as applied in the Mississippi River Basin Model [1]. Besides the downsizing from real-world to laboratory scale also the exaggeration of laboratory models has an impact on the hydraulic and/or morphodynamic system ([2]). It must be considered, that it is impossible to keep in a scaled model all relevant force ratios constant ([3]).

In the present paper the scale effects, both from nature scale to laboratory scale and the exaggeration, of a long fluvial model (Froude scaling) of the Middle Rhine with an exaggeration of n=1.2 are investigated numerically. The scale of the laboratory model is 1:50 vertical and 1:60 horizontal. To account for the highly jointed bedrock topography a new manufacturing process for the laboratory model was developed. The final laboratory model bottom consist of plane concrete parts (traditional steel profile method), highly-resolved concrete blocks (using CNC processing method), and fixed gravel material. Furthermore, additional roughness elements were introduced during the calibration of the laboratory model.

A three-dimensional hydrodynamic-numerical (3D-HN) model of the laboratory model is set up on a scale 1:1. Each surface material and structure is represented in the numerical model by an individual roughness zone characterized by an equivalent sand roughness, ks. Not all geometrical details are captured by the numerical mesh, in particular the part of the highly-resolved concrete blocks. The calibrated 3D-HN model of the laboratory river model is resized to nature-scale and to laboratory scale without exaggeration.

The underlying scaling laws are described in section II. The laboratory and numerical models applied in this study are presented in section III. In section IV the results are shown and the scale effects are analysed. The final conclusions and some ideas for further investigations are presented in section V.

II. SCALING LAWS

As mentioned above only a single force can be reproduced correctly in a scale model using the same fluid and with the same occurring gravitational force. In most river engineering purposes hydraulic laboratory models are scaled using the Froude similarity. It is applied when the dominant controlling force is gravity. The Froude scaling results in lower flow velocities in the model compared to nature. Consequently the Reynolds number is reduced, both due to the lower flow velocities and due to the smaller length scale. For a proper modelling it has to be checked that the turbulent regime (in general fully-turbulent) is the same in the model and in nature.

In Table 1 different factors for Froude and Reynolds scaling are summarized based on the geometrical scaling factor, M_L, and the exaggeration, n. For the Froude scaling it is distinguished between unexaggerated and exaggerated models. For detailed information see e.g. [4].

TABLE 1: FROUDE AND REYNOLDS SCALING FACTORS

Physical parameter		**Model law scale factor of**		
	Unit	**Froude**		**Reynolds**
		original	**exaggerated**	
Length / width	m	M_L	M_L $(= M_{Lh})$	M_L
Height	m	M_L	M_L/n $(= M_{Lv})$	M_L
Flow time, experiment duration	s	$M_L^{1/2}$	$(M_L\, n)^{1/2}$	M_L^2
Velocity	m/s	$M_L^{1/2}$	$(M_L/n)^{1/2}$	$1/M_L$
Acceleration	m/s^2	1	1/n	M_L^{-3}
Discharge	m^3/s	$M_L^{5/2}$	$(M_L^5/n^3)^{1/2}$	M_L
Relative slope	m/m	1	1/n	1

A common technique is to use different geometrical scaling factors in horizontal and vertical direction, resulting in an exaggerated model. This method makes it possible to use larger (horizontal) scaling factors, thus reducing the spatial extent of a laboratory model, without violating the Reynolds similarity. Furthermore, problems of to low water depths can be avoided, e.g. concerning the surface tension.

The exaggeration of a Froude scaled model has an impact on several hydraulic phenomena. Depending on the specific case and the amount of the exaggeration this effects might be more or less distinct. In Figure 1 three different aspects are highlighted. Exaggerated models are leading to a higher water level slope. To achieve similar velocities the bottom roughness has to be increased compared to unexaggerated nature (a). You also have to bear in mind, that vortex structures are not exaggerated in the model resulting in different proportions between the channel geometry and the vortex in the model with and without exaggeration (b). Furthermore, the exaggeration of a model results in different angles of inclination of non-horizontal structures. In the wake of these structures separations might occur resulting in higher energy losses compared to the unexaggerated model (c).

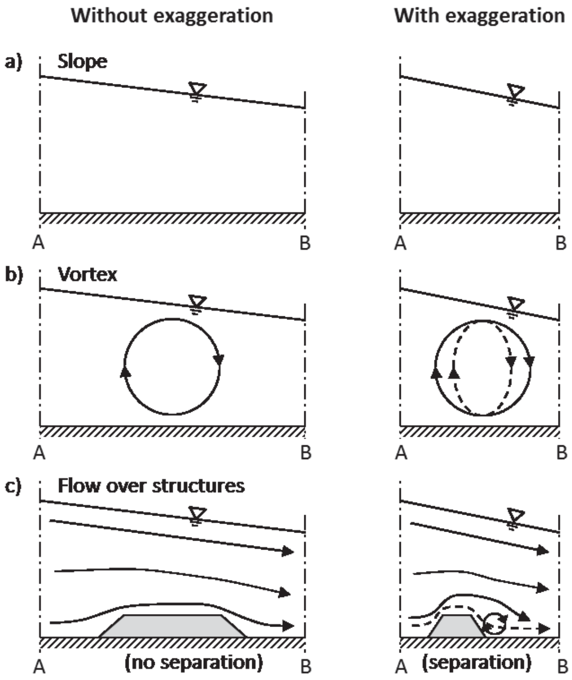

Figure 1: Influences of model exaggeration on a flume (a), a vortex (c) and an overflowed structure (d) after [5]

The roughness in scaled models is usually found by calibration. For this study the 3D-HN model of the laboratory river model is calibrated via experimentally derived roughness values from literature for the individual surface materials (concrete and gravel) and structures. For the scaled models– to nature-scale and to an unexaggerated model – two different methods are used to scale the roughness coefficient.

Firstly, the equivalent sand roughness of the Nikuradse friction law, k_s, is interpreted as a geometrical height. Thus, the same scaling as for the height is used (cf. Table 1), hereafter referred to as *geometrical scaling*.

Secondly, a scaling factor is derived based on the empirical Chézy equation

$$\bar{u} = C \cdot \sqrt{R_h \cdot S}, \qquad (2)$$

with the cross-sectional averaged velocity, $\bar{u}$, the Chézy coefficient, C, the hydraulic radius, R_h, and the bottom slope, S (for steady and uniform flow). With the assumption of a wide channel ($R_h \approx h$, with the flow depth, h) and the Froude scaling factors for an exaggerated model a scaling factor of $\sqrt{n}$ can be derived; called *Chézy scaling* in the following. For hydraulically rough flow the Colebrook-White formula correlates the Chézy coefficient and the ratio of the Nikuradse coefficient to the flow depth,

$$C = 18 \cdot \log_{10}\left(12 \cdot \frac{h}{k_s}\right). \tag{3}$$

For the scaling of the Nikuradse roughness – e.g. form the exaggerated laboratory model to nature-scale – based on the Chézy scaling and the Colebrook-White formula the flow depth must be known.

III. MODELS

A. Laboratory scale model

The laboratory model is 73 m long and represents a 4.4 km long stretch of the Middle Rhine. The central part of the investigation is a sharp 90° bend with a gravel bar on the inner bend and a rock island on the outer bend. It is designed as a long fluvial model (Froude scaling) with an exaggeration of n=1.2. The scale was defined 1:50 in vertical and 1:60 in horizontal direction. The bathymetry is presented in Figure 2.

To account for the highly jointed bedrock topography in the area of investigation a new manufacturing method was developed. By the use of CNC milling machines highly resolved concrete cast moulds were manufactured. This technique enables an extremely exact reproduction of the river bottom topography. Therefore most of the form roughness is incorporated in the model. During the calibration process different additional roughness elements were introduced (see Figure 3). For a detailed description of the laboratory model and the manufacturing process see [6].

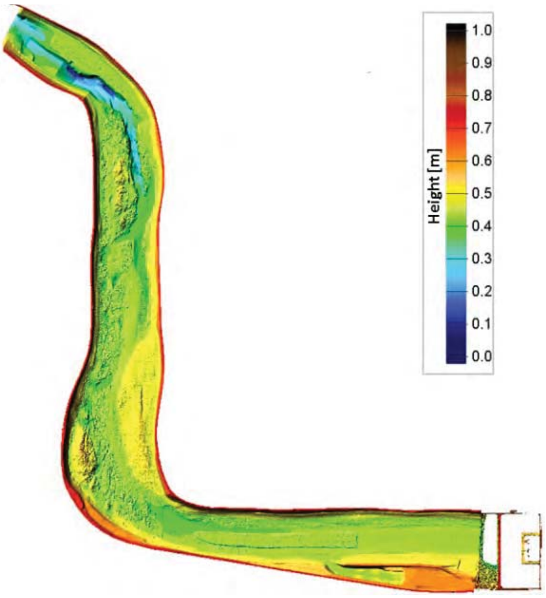

Figure 2: Bathymetry of the laboratory model

B. Numerical model

For the investigations of the scale effects three numerical models were built. The *reference model* (Lab_n12) has the same dimensions as the laboratory scale model. It is 73 m long and its bathymetry represents the laboratory model (Figure 2). Due to the used mesh size not all details of the topography are resolved, especially parts of the highly-resolved concrete blocks. This model was calibrated to velocity and water level measurements of the laboratory model. The same roughness zones as in the laboratory model (see Figure 3) were applied.

The *unexaggerated model* (Lab_n10) is the same as the reference model without the vertical exaggeration. Consequently, the slopes are the same as the slopes in nature.

The *nature-scale model* (Nat_n10) has nature dimensions, which means that the reference model is scaled by 60 in horizontal direction and by 50 in vertical direction.

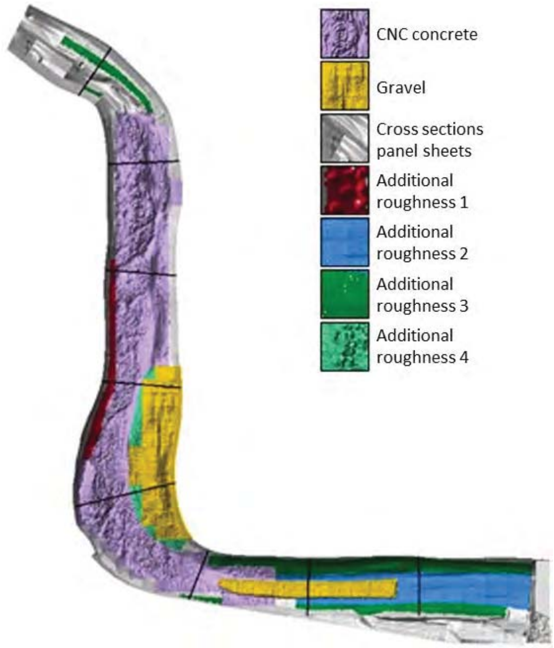

Figure 3: Roughness zones of the laboratory model

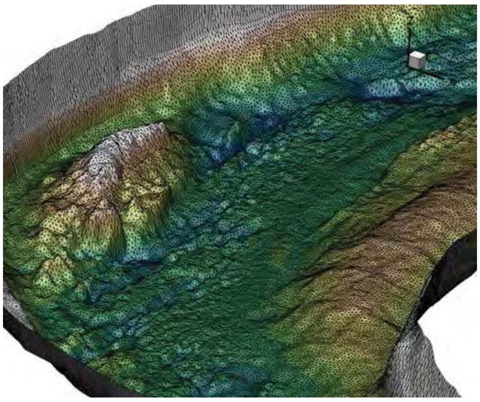

Figure 4: Grid structure of the numerical model at the 90° bend (figure exaggerated 5 times)

In Table 2 the characteristics of the three used numerical models are summarized. In the present paper all simulations were set up using stationary boundary conditions – constant volume flux at the inlet and constant water level at the outlet. For the roughness-scaling the two above mentioned methods were used: geometrical scaling and Chézy scaling. For both methods a single value per roughness zone was used (cf. Figure 3). Only results for mean water level +1 m and the corresponding discharge are shown. Further information and results for other discharges can be found in [7].

TABLE 2: CHARACTERISTICS OF USED NUMERICAL MODELS

	Reference model (Lab_n12)	Unexaggerated model (Lab_n10)	Nature-scale model (Nat_n10)
Length / width scale	1:60	1:60	1:1
Height scale	1:50	1:60	1:1
Number of nodes	26370	26370	26370
Mean horizontal node distance	44 mm (bend: 38 mm)	44 mm (bend: 38 mm)	2640 mm (b.: 2280 mm.)
Vertical discretization	15 layers	15 layers	15 layers
Roughness coefficients	Calibrated to the lab model	geometrical (1:1.2) and Chézy scaling	geometrical (1:50) and Chézy scaling

IV. RESULTS

In Figure 5 the differences of the free surface from the unexaggerated model (Lab_n10, blue line) and the nature-scale model (Nat_n19, grey line) to the reference model (Lab_n12) are shown in nature-scale. The Chezy scaling (solid lines) results in lower Nikuradse roughness values than the geometrical scaling (dotted lines). The agreement of the water levels between all three models using the Chézy scaling of the roughness is fairly good. The maximum differences are within a 5 cm range. Using the geometrical scaling of the roughness, the water level shows show higher discrepancies between the unexaggerated model (Lab_n10) and the reference model (Lab_n12).

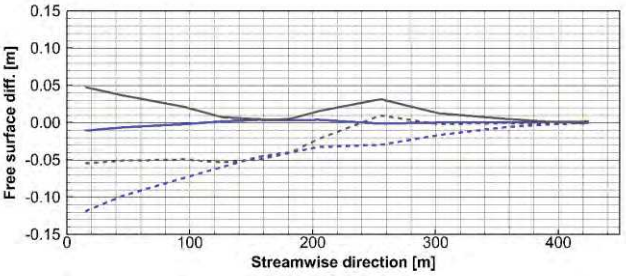

Figure 5: Differences of the free surface (blue: Lab_n10-Lab_n12, grey: Nat_n10- Lab_n12; solid: Chézy scaled, dotted: geometrical scaled

For Froude scaled models the assumption of the same laminar / turbulent regime in the prototype and in the model has to be verified. In this context a local Reynolds number is introduced,

$$Re = \frac{u \cdot h}{\nu}. \tag{4}$$

based on local depth-averaged velocity, u, the local water depth, h, and the kinematic viscosity, ν. The critical Reynolds number for open-channel flow is 2320. For laboratory investigations it is recommended to be greater than 5000 to prevent Reynolds induced scaling effects ([2]). In the following we assume a laminar flow for 0<Re<2320, a partially turbulent flow for 2320<Re<5000 and a fully-turbulent regime for Re>5000.

In Figure 6 the local Reynolds number is shown for the reference model (left) and the unexaggerated model (middle) and for the nature-scale model (right), both using the Chézy scaling for the roughness. The geometrical scaling shows almost identical results (not shown in here). In the vicinity of the 90° bend deviations in the turbulent regime can be observed at the inner bend and at the outer bend. Especially in the area of the gravel bar at the inner bend the scaling from nature-scale (Nat_n10) to laboratory scale (Lab_n12 and Lab_n10) has a crucial effect. This is due to the lower water depth in this area. In the reference model (Lab_n12) the area of laminar and partially turbulent flow is reduced compared to the unexaggerated model. The influence of the scaling effects on the flow field is evaluated in the following.

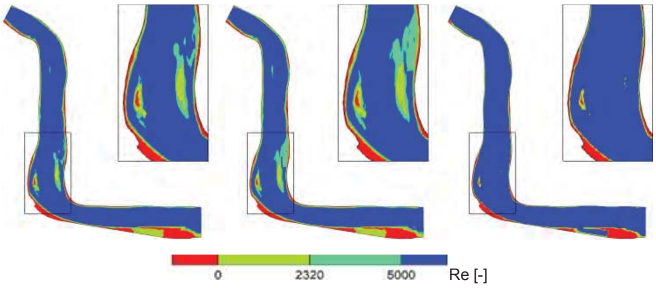

Figure 6: Local Reynolds number (left: Lab_n12, middle: Lab_n10 Chézy scaled, right: Nat_n10 Chézy scaled)

The velocity and discharge distribution is analysed at three different cross-sections for the three different models as shown in Figure 7. At the entrance of the bend (A-A) both the velocity distribution and the distribution of the specific discharge are similar in all three models (Figure 8 top). Only close to the right bank the velocities in the nature-scale model (Nat_n10, grey solid line) are smaller compared to models in laboratory scale (Lab_n10, blue solid line and Lab_n12, green solid line).

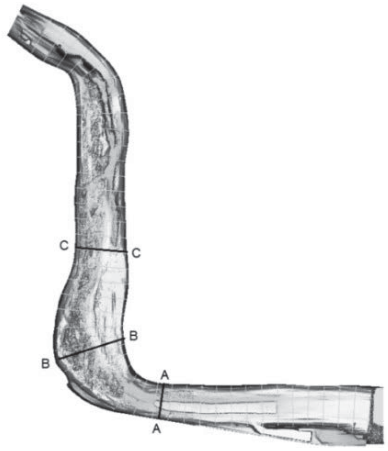

Figure 7: Cross-sections before (A-A), in (B-B) and after (C-C) the 90° bend

The distributions of the specific discharge show also in the bend (B-B) a good matching between the models (Figure 8 middle). The depth-averaged velocity shows differences in the range of the inner bend gravel bar. At a distance from 300 to 440 m to the left bank the flow velocities are in the nature-scale model (Nat_n10, grey solid line) significantly lower than in the models in laboratory scale (Lab_n10, blue solid line and Lab_n12, green solid line). Furthermore, in this part the distribution of the depth-averaged velocity differs, whereas the specific discharge (dashed lines) is similar in all models. With 70 % the maximum relative deviation of the specific discharge occurs at around 370 m to the left bank.

After the bend (C-C) both the velocity and discharge distribution show a slightly different distribution between the left and the right part of the channel (Figure 8 bottom). In the nature-scale model (Nat_n10, grey dashed line) the discharge is higher (up to 13 %) in the left part than in the models with laboratory scale and lower in the right part (up to 50 %). The reference model (Lab_n12, green lines) shows less deviation from the nature-scale model (Nat_n10, grey lines) than the unexaggerated model (Lab_n10, blue lines). This behaviour is similar to the differences of the laminar / turbulent regime as shown above in Figure 6. Violating the requirement of a fully turbulent regime, especially in the area of the gravel bar, leads to a less exact reproduction of the discharge distribution.

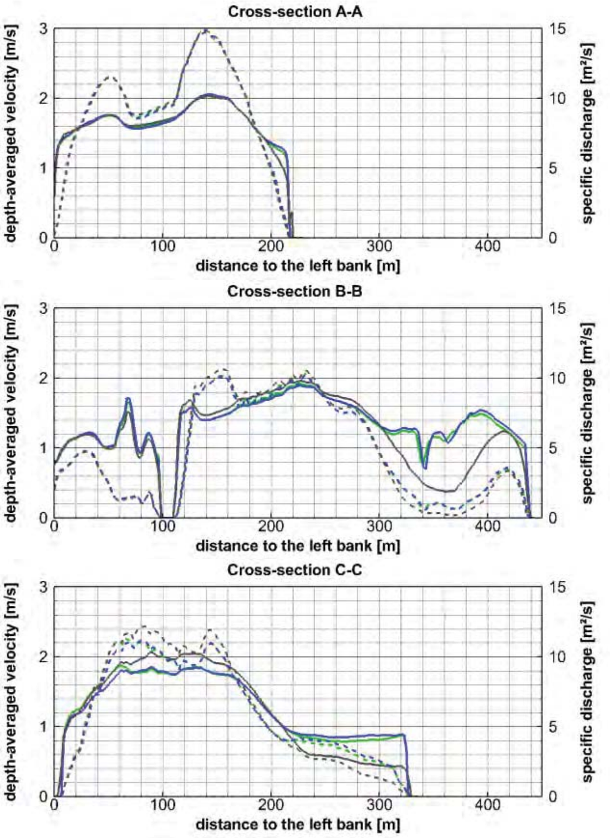

Figure 8: Depth-averaged velocity (solid) and specific discharge (dashed) distribution in three cross-sections (top: A-A, middle: B-B, bottom: C-C) for all three models (green: Lab_n12, blue: Lab_n10 Chézy scaled, grey: Nat_n10 Chézy scaled)

The differences of the specific discharge between the geometrical scaling and the Chézy scaling of the roughness are shown in Figure 9, both for the unexaggerated model (Lab_n10, blue lines) and the nature-scale model (Nat_n10, grey lines). The maximum deviations are in the order of 10 %, except in regions with very low specific discharges. In the cross-sections B-B and C-C the differences between nature-scale model and the two laboratory-scale models at least on order of magnitude larger than the differences between the two roughness scaling methods. Thus, in the specific case the influence of the roughness scaling is smaller than the geometrical (length-) scaling with a factor of 60.

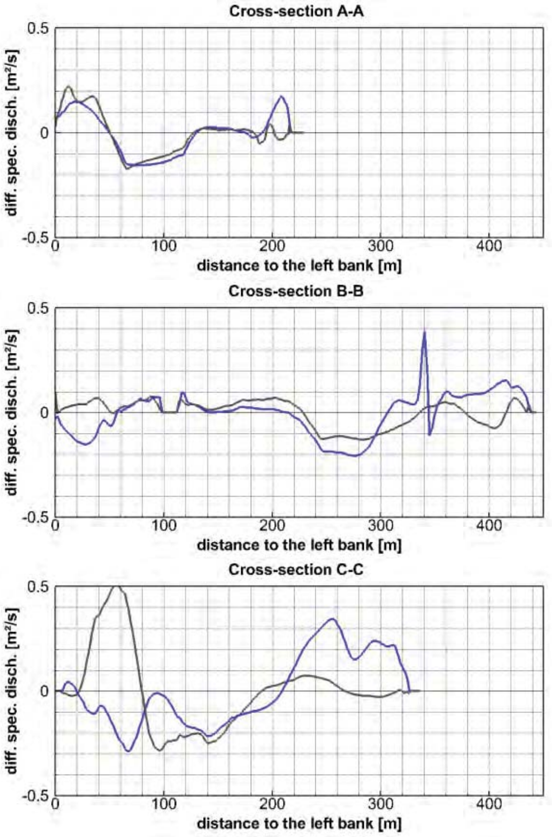

Figure 9: Differences of the specific discharge between the Chézy scaled roughness and the geometrical scaled roughness (blue: Lab_n10, grey: Nat_n10)

Beside the distribution of the velocity and the discharge the secondary flow in the 90° bend plays a crucial role for the evaluation of the scaling effects. In this context streamlines close to the bottom (near-bottom), at the height of the first cell, and on the free surface are compared. The starting line for the streamlines is located above the 90° bend slightly left of the channel centre line. This location is chosen as it is potentially on the main path of the sediments depositing at the inner bend.

In Figure 10 the streamlines in the nature-scale model are shown based on the depth-averaged velocity, the velocity at the free surface and the near-bottom velocity. Due to the occurring secondary flow induced by the channel curvature the fluid close to the bottom flows in the direction of the inner bend and at the free surface in direction of the outer bend. This separation shows the highly three-dimensional character of the occurring flow field at the 90° bend section.

Figure 10: Streamlines nature-scale model, Nat_n10 Chézy scaled (grey: depth-averaged velocity, green: velocity at the free surface, blue: near-bottom velocity)

In Figure 11 the near-bottom streamlines and the streamlines at the free surface are compared for the nature-scale model (Nat_n10, black lines) and the reference model (Lab_n12, blue lines). In the zoom on the channel bend very small differences can be observed. In the nature-scale model the effect of secondary flow seems to be slightly higher. Due to the small differences the streamlines for the unexaggerated model (Lab_n10) are not shown here.

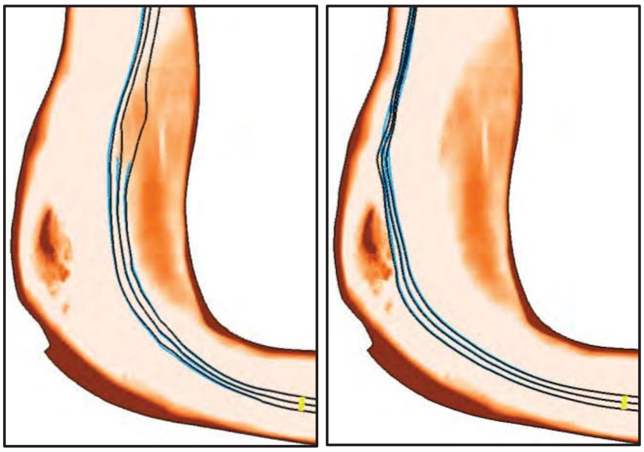

Figure 11: Left: near-bottom streamlines, right: free-surface streamlines (blue: Lab_n12 Chézy scaled, black: Nat_n10 Chézy scaled)

Despite the discrepancies of the velocity and discharge distribution the secondary flow effect on the separation of the flow field is only slightly affected. Due to the very small differences between the two roughness-scaling methods, the streamlines for the geometrical scaling case are not shown in here.

V. CONCLUSIONS AND OUTLOOK

The influence of scaling effects using laboratory scale models are highlighted and analysed for a specific laboratory model of the Middle Rhine. Due to scaling limitations using the same fluid (water) in the laboratory model and in nature and the same gravitational force, scaling effects are always occurring.

In the present case both the velocity distribution and the distribution of the specific discharge is affected by not fully ensuring Reynolds similarity. At the inner bend partially not fully-turbulent flow occurs at laboratory scale due to the low water depth in this part. This limitation is also not compensated by the used exaggeration factor of 1.2 – even though the results show less deviation to the nature-scale model than the unexaggerated model. Regarding the important effect of the secondary flow – resulting in a separation of the near-bottom flow and the flow at the free surface – only slight deviations between the nature-scale model and the reference model can be observed.

The two roughness-scaling methods showed only slight differences in the discharge distribution in the three cross-sections. The roughness-scaling based on the Chézy equation showed for the presented discharge less deviations of the water level compared to the reference case. The Chézy-scaling method is recommended for further investigations.

Depending on the problem formulation and the area of interest the laboratory scale models show good agreement to the investigated nature-scale model. The exaggeration of the model seems to have lower impact than the scaling from nature-scale to laboratory scale, which is the basis for laboratory investigations.

For future studies it is planned to perform a scale series to investigate the scaling effects with increasing / decreasing horizontal and vertical scaling factors. The aim is to identify a critical exaggeration factor which might significantly influence results.

In general, for laboratory investigations in river engineering it is of great importance to understand the influence of the occurring scaling effects. The presented method shows a cost-effective approach and the great value of hybrid modelling while working on river engineering challenges.

Nevertheless one has to bear in mind that the current investigation is performed only numerically. The influence of possible effects originated from this method, like numerical diffusion, was not evaluated in detail in the present study. It is assumed that the findings presented in this paper are not affected crucially by these artefacts, because all conclusions are based on comparative analysis.

REFERENCES

[1] K. D. Cheramie, “The Scale of Nature: Modeling the Mississippi River”, Places Journal.

[2] H. Dorer, “Similitude of Fluvial Models” (in German), Mitteilungsblatt der BAW, 54, 1984.

[3] V. Heller, “Scale effect in physical hydraulic engineering models”, J. Hydr. Res., Vol. 49, No. 3, 2001, pp. 293-306.

[4] H. Martin and R. Pohl, “Technical hydromechanics” (in German), Verlag für Bauwesen, Berlin, 2000.

[5] DVWK 1984, “Wasserbauliches Versuchswesen”, Heft 39.

[6] S. Wurms, T. Hüsener, F. Folke, N. P. Huber and A. Schmidt, “Growing challenges in river engineering – What contribution can hybrid modeling make?”, E-proceedings of the 2nd International Symposium on Hydraulic Modeling and Measuring Technology Congress, Nanjing, China, 2018.

[7] A. Haehne, “Numerical Modeling (3D-HN) of a highly resolved laboratory model in laboratory scale and nature-scale” (in German), Master Thesis, KIT Karlsruhe, 2018.