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A Comparison between Inclined Hydraulic Jumps over a Thin Wall and a Step

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ABSTRACT: In this study, which is based on previous experimental data by the authors, a comparison is attempted between inclined hydraulic jumps over a thin wall, and a step - which is followed by a raised floor. The water free surface profiles, the dimensionless lengths and dimensionless conjugate depths are examined and the results are compared and discussed. The two flow cases appear to have some similarities between the flow lengths, but they present considerable differences in the descending part of the water free surface profiles and the conjugate depths. Some explanations are offered in order to compare the two flow structures. The results of this investigation may be useful to the hydraulic engineer when designing the pertinent open channel — especially for low head-works.

Keywords: Hydraulic Jumps, Thin Walls, Abrupt Steps, Low Head.

1 INTRODUCTION

The stable horizontal or inclined (angle $\phi$) water flows over a thin wall ($w$), perpendicular to the channel floor, and over a step ($w$) - which has its upstream face perpendicular to the channel floor, are two interesting and comparable hydraulic problems, where the two flow obstacles are often called “sills”. In both cases an incipient hydraulic jump is present, which is stable and may be used to dissipate the jump’s mechanical excessive energy.  

Figs. 1 and 2 show the basic flow characteristics of the two water flows, within the inclined rectangular channel. Both structures are mainly linked to rather low water heads because of their natural characteristics, where the low heads are important here.
In both jump cases—of rather low head—there are three important flow cross sections: Section 1 (of uniform flow) with depths $d_1$ (Fig. 1) and $d_m$ (Fig. 2), and Section 2 (of uniform flow) with depths $d_2$ (Fig. 1) and $h_m$ (or $d_m$), (Fig. 2), where $h_m=d_m+w$ and $h_2=d_2+w$. At a distance $x$ the flow depth is $d$ (Fig. 1) and $h$ (or $d$), Fig. 2, while at distance $x'$ (from section m) corresponding depths are $d'$ (Fig. 1) and $h'$ or $d'$ (Fig. 2). The respective lengths (Figs. 1 and 2) are $L_1$, $L_2$, with $L=L_1+L_2$, the discharge (per unit channel width) is $q$, while in the case of the jump over a step an air pocket may be present, which has not been observed in the case of the jump over a thin wall. The max depths ($d_m$ in Fig. 1 and $h_m$ in Fig. 2) may appear close upstream, on, or usually—close downstream the wall and the step face correspondingly.

The main flow parameter is the Froude number at section 1, 

$$Fr_1 = \frac{q \cdot g^{1/2} \cdot d_1^{3/2}}{d_1},$$

which is bigger than 1, since in both cases the upstream flow is supercritical, while the downstream flow ends up (here) to a channel drop.

Demetriou et al (2008), and (2010), have performed a large number of experimental measurements for flows over thin walls, Demetriou et al (2010), are also dealing with the jump over a step, while Hager (1992), is presenting a wide summary of both flows in exclusively horizontal channels. For the jumps over a thin wall the measurements are extended to $0^\circ \leq \phi \leq 14^\circ$, while for the jumps over a step it is $0^\circ \leq \phi \leq 8^\circ$.

All laboratory Runs were twice performed in a rather small perspex flume, suitable for non uniform flow phenomena measurements.

2 PREVIOUS DATA

2.1 Flow Profiles

2.1.1 Jumps Over Thin Walls

In order to determine the surface water profiles, $d(x)$ and $d'(x')$, the dimensionless terms

$$\bar{d} = \frac{(d-d_1)}{(d_m-d_1)} \text{ vs } \bar{x} = \frac{x}{L_1},$$

$$\bar{d}' = \frac{(d_m-d_1')}{(d_m-d_2)} \text{ vs } \bar{x}' = \frac{x'}{L_2}$$

are used for upstream and downstream profiles, while $d_m+w$ are the water hydraulic heads.

The experimental results gave the profiles’ empirical equations, Demetriou et al (2008),

$$\bar{d} \approx \left(7.48 - 4.76 \cdot e^{J_o} \cdot (\bar{x}) - (6.48 - 4.76 \cdot e^{J_o}) \cdot (\bar{x})^{1.5}\right),$$

where $J_o$=channel slope=$\sin \phi$ and $0^\circ \leq \phi \leq 14^\circ$, $0.14 \leq d_1/w \leq 1$, $2 \leq Fr_1 \leq 7.2$, and

$$\bar{d} \approx a \cdot (\bar{x})^{1.5} - b \cdot (\bar{x})^3$$

where $a$, $b$, are arithmetic coefficients, with particular values for each angle $\phi$. The approximate values of $a$, $b$, are presented in Table 1. In these profiles, for $\bar{x} = 0$ it is $\bar{d} = 0$ (or $d=d_1$) and for $\bar{x} = 1$ it is $\bar{d} = 1$ (or $d=d_m$), while for $\bar{x} = 1$ it is $\bar{d}' = 1$ (or $d'=d_2$) and for $\bar{x} = 0$ it is $d' = d_m$. 

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Table 1. Arithmetic coefficients.

<table>
<thead>
<tr>
<th>φ</th>
<th>a</th>
<th>b</th>
<th>(a-b)</th>
</tr>
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<tr>
<td>0°</td>
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<td>0.91</td>
<td>1.02</td>
</tr>
<tr>
<td>2°</td>
<td>2.04</td>
<td>1.03</td>
<td>1.01</td>
</tr>
<tr>
<td>4°</td>
<td>2.14</td>
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<tr>
<td>6°</td>
<td>2.16</td>
<td>1.14</td>
<td>1.02</td>
</tr>
<tr>
<td>8°</td>
<td>2.26</td>
<td>1.23</td>
<td>1.01</td>
</tr>
</tbody>
</table>

Eq. (2)

2.1.2 Jumps Over Steps
The corresponding water profiles’ dimensionless terms are, Demetriou et al (2010),
\[ h = \frac{(h - h_1)}{(h_m - h_1)} \text{ vs } x = x / L_1, \]
\[ h' = \frac{(h_m - h')}{(h_m - h_2)} \text{ vs } x' = x' / L_2 \]
while the water hydraulic heads are \( h_m \)-w.

The experimental results gave the profiles’ empirical equations (φ in degrees)
\[ h \approx (2.85 - 0.09 \cdot \varphi) \cdot x - (1.85 - 0.09 \cdot \varphi) \cdot (\bar{x})^{1.5} \quad (3) \]
\[ h' \approx 4.48 \cdot (x)^{1.5} - 3.48 \cdot (x)^2 \quad (4) \]
i.e. the downstream profile is a unique curve – independent of angle φ. Actually, there are various profiles, but they are so dense (within the present flow parameters) that they can be represented by a single line. The above equations hold for \( 3 \leq h_m / h_1 \leq 31 \), \( 1.5 \leq h_m / h_2 \leq 5 \), \( 0.2 \leq h_1 / w \leq 6.7 \) and \( 1.1 \leq Fr_1 \leq 9.5 \). In these profiles, for \( \bar{x} = 0 \) it is \( h = h_1 \) and for \( \bar{x} = 1 \) it is \( h = h_m \), while for \( \bar{x}' = 0 \) it is \( h' = h_m \) and for \( \bar{x}' = 1 \) it is \( h' = h_2 \).

2.2 Jumps’ Lengths
2.2.1 Jumps Over Thin Walls
The dimensionless lengths \((L_1-w)/d_m\) vs \( Fr_1 \) and angle \( \varphi \) (\( 0^o \leq \varphi \leq 14^o \)) were measured by Demetriou (2010), for \( 0.14 \leq d_1 / w \leq 1 \) and \( 2 \leq Fr_1 \leq 7.2 \). These results were suitably adjusted here in order to compare with the present results for the jumps over steps.

2.2.2 Jumps Over Steps
Demetriou et al (2010), have experimentally determined the dimensionless lengths \((L_1-w)/d_1\) vs \( Fr_1 \) and angles \( \varphi \) (\( 0^o \leq \varphi \leq 8^o \)), while they have also measured \( h_m / d_1 \) vs \( Fr_1 \) and angles \( \varphi \) (same range). Thus, dividing \((L_1-w)/d_1\) to \( h_m / d_1 \) the ratio \((L_1-w)/h_m\) can be received.

2.3 Jumps’ Conjugate Depths
In a wide view, \( d_m \) (Fig. 1) and \( h_m \) (or \( d_m \)) – Fig. 2, may be considered as conjugate depths in relation to \( d_1 \) and \( h_1 \) (=d_1) respectively.

2.3.1 Jumps Over Thin Walls
Demetriou et al (2010), have measured \((d_m-w)/d_1\) vs \( Fr_1 \) (\( 1.5 \leq Fr_1 \leq 7.2 \)) and angle \( \varphi \) (\( 0^o \leq \varphi \leq 14^o \)). These results are slightly modified here in order to compare with the corresponding quantities of the jumps over steps.

2.3.2 Jumps Over Steps
Demetriou et al (2010), have also measured \((h_m-w)/d_1\) vs \( Fr_1 \) (\( 2 \leq Fr_1 \leq 8.5 \)) and angle \( \varphi \) (\( 0^o \leq \varphi \leq 8^o \)).
3 COMPARISONS. RESULTS AND DISCUSSION

The comparison between the geometrical elements of both jumps is restricted here to corresponding water free surface profiles along the entire lengths $L$, the dimensionless lengths along the ascending parts of these profiles, $L_1$, and the conjugate depths ratios.

Figs. 3, 4 present the flow profiles for the jumps over a thin wall and over a step correspondingly, while Fig. 5 presents a comparison between these profiles, with solid lines two limit profiles for $\varphi=0^\circ$ and $\varphi=14^\circ$ (jumps over walls) and with dashed lines two limit profiles (jumps over steps) for $\varphi=0^\circ$ and $\varphi=8^\circ$. The use of only two couples of limiting profiles in Fig. 5 (and not of all profiles – as in Fig. 3 and 4) is due to the narrow space in this figure.

As it is clear from Fig. 5, along $L_1$ the ascending flow profiles may compare among them, showing that from this point of view the two jumps, over a thin wall and a step, appear to have a similar structure. On the contrary, along $L_2$ the descending free surface water profiles are clearly different in the two flow phenomena: Just after the thin wall the water is dropping down, mainly because of gravity, while after the step the flow is continued along the raised floor. Perhaps this behavioral difference is also responsible for the existence or no of an air pocket in the two flow cases.

![Figure 3](image1.png)  
**Figure 3.** Flow profiles for jumps over a thin wall.

![Figure 4](image2.png)  
**Figure 4.** Flow profiles for jumps over a step.

![Figure 5](image3.png)  
**Figure 5.** Comparison of flow profiles for jumps over a thin wall and over a step.
Fig. 6 presents the dimensionless lengths \((L_1 - w)/d_m\) vs \(Fr_1\) and angle \(\phi\) for the jump over a thin wall, Fig. 7 shows the lengths \((L_1 - w)/h_m\) vs \(Fr_1\) and angle \(\phi\), for the jumps over a step, while Fig. 8 presents a comparison between corresponding lengths, where for \(w \to 0\) (no wall and no step) \(d_m \to d_1\) (Fig. 1), \(h_m \to h_1\) (Fig. 2) and \(L_1 \to 0\), i.e. both families of lines tend to the infinity.

![Figure 6](image-url)

**Figure 6.** \((L_1 - w)/d_m\) vs \(Fr_1\) and angle \(\phi\) for jumps over a thin wall.

![Figure 7](image-url)

**Figure 7.** \((L_1 - w)/h_m\) vs \(Fr_1\) and angle \(\phi\) for jump over a step.

![Figure 8](image-url)

**Figure 8.** Comparison of flow lengths for jumps over a thin wall and over a step.

From Fig. 8 it is clear that, although any wall length curves (solid lines) lie over corresponding step – length curves (dashed lines), the two families of lines are similar and almost of the same order of magnitude. From this point of view both jumps (over a thin wall and over a step) appear to have a comparable structure.

Next figures compare the dimensionless conjugate depths for both inclines jumps.

Fig. 9 shows a diagram between \((d_m - w)/d_1\) vs \(Fr_1\) and angles \(\phi\) for jumps over a thin wall, Fig. 10 presents \(d_m/h_f=(h_m - w)/h_1\) vs \(Fr_1\) and angle \(\phi\) for jumps over a step, while Fig. 11 attempts a comparison between corresponding quantities of the two jumps (solid lines for jumps over a thin wall and dashed lines for jumps over a step).
For $w \to 0$ (no wall and step) $d_m \to d_1$ (or $d_m/d_1 \to 1$) and $h_m \to h_1$ (or $h_m/h_1 \to 1$), i.e. the horizontal line (Fig. 11) through $(d_m - w)/d_1$ or $(h_m - w)/h_1$ is the lower limit of all curves of Fig. 11, independently of angle $\phi$ and $F_{r1}$.

As it clear from Fig. 11, for same $w$ and initial depths $(d_1, h_1)$ the jump over a step appears to have much larger conjugate depths than respective jump over a thin wall (for same angle $\phi$ and $F_{r1}$). Especially for larger angles $\phi$ the dimensionless conjugate depth is enormously larger in the case of jumps over steps than in the case of jumps over thin walls.

The above behavior shows that the presence of a step (and following raised floor) produces a much larger effect on the flow, than the presence of a thin wall – especially in larger angles $\phi$ and Froude numbers $F_{r1}$. If this result is combined with the profiles’ result of Fig. 5 – along $L_2$, then the final conclusion may be that the raised floor (in the case of the jump over a step) makes the main difference of the two flow cases. This difference is primarily expressed in the quantities which are perpendicular to the flow direction (conjugate depths, $d'$, $h'$) and not parallel to the flow (jumps’ dimensionless lengths).

Figure 9. $(d_m - w)/d_1$ vs $F_{r1}$ and angle $\phi$, over a thin wall.

Figure 10. $(h_m - w)/h_1$ vs $F_{r1}$ and angle $\phi$, over a step.

Figure 11. Comparison of conjugate depths for jumps over a thin wall and over a step.
In Figs. 9, 10 and 11 the dimensionless water heads \((d_{m\cdot w})/d_1\) and \((h_{m\cdot w})/d_1\) appear to have small or larger values, depending on \(Fr_1\), angle \(\phi\) and \(d_1\). The last depth is quite small and this is why some large dimensionless water heads are present in these Figs. Actually all heads \((d_{m\cdot w})\) and \((h_{m\cdot w})\) are low when compared to \(w\), for \(0^\circ \leq \phi \leq 8^\circ\) and \(2\leq Fr_1 \leq 7\), as they were preliminary measured by the authors.

4 CONCLUSIONS

In this study, which is based on previous experimental data by the author et al, a comparison is attempted between inclined (angle \(\phi\)) hydraulic jumps over a thin wall \((w)\) and a step \((w)\). The water free surface profiles, the dimensionless lengths and dimensionless conjugate depths are compared in order to show the similarities and differences of the two flow cases. The main conclusions are: (1) The ascending parts of the water free surface profiles appear to have a similar behavior in structure and order of magnitude. (2) The descending parts of the water free surface profiles present considerable differences between them. (3) These differences are mainly due to the raised floor – following the flow over the steps. (4) The dimensionless lengths of both flows are comparable between them. (5) The dimensionless conjugate depths of both flows are quite different in magnitude. (6) The latter difference appears as a result of the raised floor – following the jump over a step. (7) The difference between the two flow cases is primarily expressed in the quantities which are perpendicular to the floor, while the quantities along the flow are almost of a similar behaviour. The results of this investigation may be useful to the hydraulic engineer when designing pertinent open channel works-especially in lower water heads cases.

NOTATION

- \(g\): discharge per unit channel width
- \(d\): gravity acceleration
- \(d\): water profile and specific depths
- \(h\): water profile and specific depths
- \(x\): longitudinal coordinate
- \(\phi\): channel angle (degrees)
- \(J_o\): channel slope
- \(Fr\): Froude number
- \(\tilde{x}\): dimensionless longitudinal coordinate
- \(\tilde{d}\): dimensionless water profile
- \(\tilde{h}\): dimensionless water profile

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