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Surface and subsurface flows: a coupled approach using ESTEL and TELEMAC2D

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***Abstract*— Surface and subsurface flow systems are inherently unified systems that are often broken into sections for logical and technical reasons. For instance, in the TELEMAC-MASCARET suite, distinct codes deal with surface (MASCARET, TELMAC2D and TELEMAC3D) and subsurface (ESTEL) flows.**

While in most applications, such decoupling of the systems works well and allows a very accurate and efficient description of the individual system by treating the adjacent system as a constant boundary condition, in the case of water flow over a porous medium, it is not adequate. Thus, in order to improve the accuracy of simulated overland flows, it is now often required to integrate the interaction between surface and subsurface flows (or free surface and groundwater flows). For now, this interaction is simulated using TELEMAC2D for free-surface flows and the ESTEL (EDF R&D) for variably saturated groundwater flows.

One major difficulty of coupled surface and subsurface simulations is to treat the evolution of the coupling interface in respect to the height of the river. Thus, the coupling strategy should be robust and generic enough to manage this problem. In this study, we present our latest developments and results on academic and real coupled surface-subsurface flows. The governing equations, the considered assumptions for each code and the details of the iterative coupling procedure are presented.

I. Introduction

Free surface flows and groundwater flows are generally simulated separately and industrial applications often neglect or oversimplify their interactions. The main reason is that the spatial and time scales of their flows are largely different. In this case, simulations of surface flows usually treat the impacts of the groundwater table as steady boundary conditions. Only few coupled strategies can be found [1] and two different approaches are used: the fully coupled approach [2] and the iterative coupling [3]. The iterative coupling seems to be more adapted to industrial applications, as the codes are used separately but communicate throughout boundary conditions [4]. In this paper, we describe in details our coupling strategy in Section II. The error assessment of the coupling is presented in Section III. Finally, in Section IV, the exchanges between the river and the groundwater table are quantified on a simple river case.

II. Description of the coupling approach

Our coupling strategy is based on the use of the two codes ESTEL and TELEMAC2D. They respectively deal with the groundwater and shallow water flows.

A. TELEMAC2D

TELEMAC2D is an open source code which belongs to the OpenTELEMAC-MASCARET platform. It is based on the shallow water equations that can be solved with the finite element method or the finite volume method.

The shallow water equations can be obtained by deriving a depth integrating form of the Navier-Stokes equations. Moreover, some assumptions are to be made:

- Horizontal length scale must be greater than the vertical one.
- The fluid is considered to be incompressible with a constant density.
- The horizontal velocity field is constant throughout the depth of the fluid.
- The pressure is hydrostatic.
- The vertical acceleration is neglected.

Shallow water equations consist of the continuity equation:

$$\frac{\partial h}{\partial t} + div(\vec{Q}) = Sce \quad (1)$$

And the momentum equations:

$$\frac{\partial Q_x}{\partial t} + div(Q_x\vec{u}) = -gh\frac{\partial Z_s}{\partial x} + hF_x + div\left(hv_e\overrightarrow{grad}(u)\right) \quad (2)$$

$$\frac{\partial Q_y}{\partial t} + div(Q_y\vec{u}) = -gh\frac{\partial Z_s}{\partial y} + hF_y + div\left(hv_e\overrightarrow{grad}(v)\right) \quad (3)$$

Where:

- h is the water height (m),
- $\vec{Q} = h\vec{u}$ is the discharge per length unit ($m^2.s^{-1}$),
- u and v are the components of the velocity vector $\vec{u}$ ($m.s^{-1}$),

- g is the gravitational acceleration ($m.s^{-2}$),
- v_e is the effective diffusion which models turbulent viscosity and dispersion ($m^2.s^{-1}$),
- Z_s is the elevation of the free surface (m),
- t is the time (s),
- Sce is the source term ($m.s^{-1}$) which has to be multiplied by the surface to obtain a discharge.

The momentum equations can be decomposed in five terms, which can be related to:

- The local flow acceleration,
- The advection term,
- The free surface gradient term,
- The source term,
- The diffusion term.

B. ESTEL

ESTEL is a proprietary code of EDF, which uses several libraries of OpenTELEMAC-MASCARET, such as the finite element solver BIEF and the MPI based module PARTEL. This code is based on the Richards equation, which can be obtained by combining the continuity equation with the Darcy law. This equation allows modeling the motion of the water in saturated and unsaturated soils:

$$C(h,\theta)\frac{\partial h}{\partial t} = div(k_r(h,\theta)K_s\overrightarrow{\mathrm{grad}}(h+z)) + Q_s \qquad (4)$$

Where:

- k_r is the relative hydraulic conductivity (-),
- K_s is the saturated hydraulic conductivity ($m.s^{-1}$),
- θ is the water content (-),
- Q_s is the volumetric source term (s^{-1}),
- h is the pressure head (m),
- $C(h,\theta) = \frac{\partial \theta}{\partial h}$ is the soil capacity (m^{-1}).

The soil capacity is determined thanks to the empirical Van Genuchten model:

$$\left.\begin{aligned} \theta &= \begin{cases} \theta_s + Se(\theta_s - \theta_r) & if\ h < 0 \\ \theta_s & if\ h \geq 0 \end{cases} \\ Se &= \begin{cases} \frac{1}{(1+|\alpha h|^n)^m} & if\ h < 0 \\ 1 & if\ h \geq 0 \end{cases} \\ k_r &= \begin{cases} Se^L(1-\left(1-Se^{1/m}\right))^2 & if\ h < 0 \\ 1 & if\ h \geq 0 \end{cases} \end{aligned}\right] \qquad (5)$$

Where L, n, m, α, θ_s and θ_r are the parameters which depend on the properties of the soil.

It is important to remind that, in this form, the Richards equation is non-conservative, which implies that mass balance errors are generally non negligible. However, we chose to use this form as it allows managing both saturated and unsaturated soils [5]. Moreover, due to an intrinsic limitation of ESTEL, only tetrahedron meshes can be used.

C. The coupling strategy

As we previously mentioned, our coupling strategy is based on the use of two separate modules interacting throughout an iterative process. Thanks to this strategy, we avoid to solve a larger system of equations (shallow water and Richards equations) with different mathematical properties. Furthermore, the iterative coupling is easier to implement and preserve the independence of the two codes. In this case, the communication is done throughout boundary conditions.

For a given time step, the coupling strategy can be decomposed into 4 steps (see Fig. 1):

Step 1 – During the initialisation step, the intersection between the river and the ground is determined. The part of the ground below the river is set to have a Dirichlet boundary conditions on the pressure head, while the rest of the surface is set with a no-flux boundary conditions. The value of the boundary conditions on the pressure for ESTEL and the source term for TELEMAC2D are initialized to the value of the previous iterations.

Step 2 - TELEMAC2D computes the solution of the shallow water equations with the enforced flux q handled as a source term. The river height is sent to ESTEL.

Step 3 – ESTEL computes the solutions of the Richards equation with the enforced boundary conditions on the pressure head h. Thus, the flux at the interface between the river and the ground is deduced from the pressure field thanks to the Darcy law. The flux q is then sent to TELEMAC2D.

Step 4 – The evolution of the river height is computed. If the value is grower than the criteria, a new coupling iteration is started by returning to step 2. Otherwise the coupling iteration is finished and a new time step is started.

It is important to notice that the coupling strategy is based on the assumption of a continuous pressure head at the interface. Thus, it is mandatory to define coherent initial conditions to conserve the numerical stability. Moreover, the runoff effect (the flow of water that occurs when the water table is higher than the river) is neglected. This simplification implies an overestimation of the exchange at the interface between the river and the ground, to counterbalance the lack of runoff [6]. It is a drawback of our coupling strategy in which exchanges can only occur at the surface of the ground covered by the river. Finally, with coupling strategy, the solver based on the finite volume method cannot be used in TELEMAC2D. In fact, as the time step is governed by ESTEL, it cannot be adapted to the CFL required by the FVM.

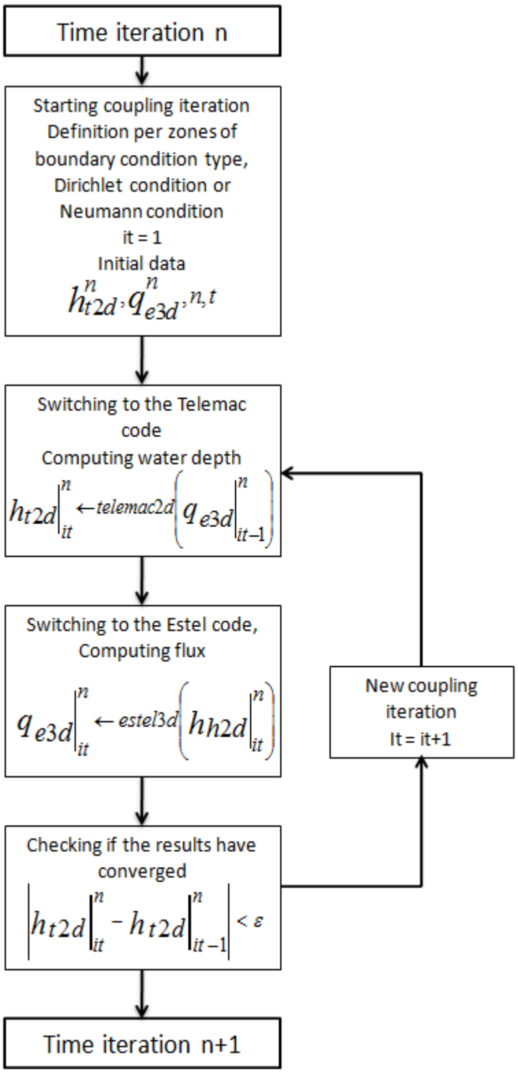

Figure 1. Coupling scheme

III. PSEUDO ARTESIAN WELL CASE

A. Configurations

This case is inspired from the Artesian well case that can be found in [3], where the water is transferred form a saturated soil to a well until the hydrostatic equilibrium is reached. We decided to use a sloping boundary (originally vertical) between the surface water and the ground (see Fig. 2) in order to validate the transition of the boundary conditions (form Dirichlet to Neumann and reciprocally) with respect to the water height.

An example of the mesh is presented in Fig. 2. The dimensions of the rectangular basis are 30*90 meters and the slope begins at 10 meters and finishes at 40 meters high. The simulation is initialized to a fully saturated ground up to 30 meters and the surface water is at rest. All the boundary conditions (except those at the coupling interface) are set to no-flux. In order to model a loamy sand [7], the parameters of the Van Genuchten model were set to $\alpha = 0.2$, $n = 1.56$, $\theta_s = 0.37$, $\theta_r = 0.17$, $L = 0.5$, $m = 0.359$ and K the hydraulic conductivity was set equal to 10^{-4} $m.s^{-1}$.

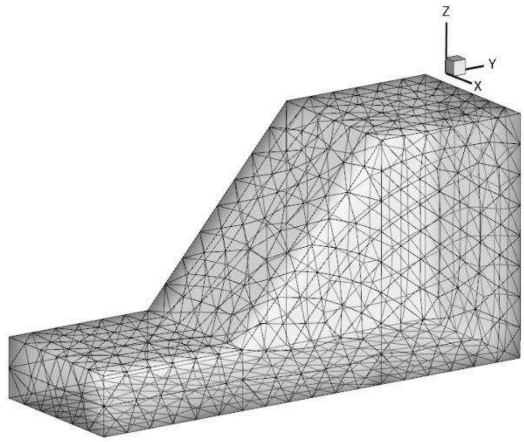

Figure 2. Artesian well case

The goal of this test case is to quantify the error that can be obtained with our coupling strategy. To do so, the final volume of water (in TELEMAC2D and ESTEL) is compared to the initial one. We can use this simple strategy to quantify the error as there is no income or outcome of water in this case.

B. Results

In order to quantify the error related to the coupling strategy, we decided to study the loss of mass with respect to the initial difference between the groundwater table and the river height. In all simulations, the water table is initialized to a 30 meters height, which corresponds to an initial volume of water of 24566 m^3. The well height is initialized from 20 meters height (i.e. 10 meters below the water table) up to 30 meters height (same height as the water table), for an initial volume of water from 10535 to 20026 m^3. The error (i.e. loss of mass) with respect to the difference between the water table and the well height are presented in Tab. 1. For every simulation, the error is decomposed in three parts: the loss of water due to TELEMAC2D, ESTEL and the coupling.

TABLE I. COMPARISON OF THE DIFFERENTS ERRORS AT THE END OF THE SIMULATION

δh in m (water table level above the river level)	TELEMAC2D error in m^3 lost	ESTEL error in m^3 lost	Coupling error in m^3 lost
0.5	2.4×10^{-10}	4.1×10^{1}	-2.4×10^{-10}
1	4.5×10^{-10}	8.6×10^{1}	-4.5×10^{-10}
1.5	1.8×10^{-9}	1.3×10^{2}	-1.8×10^{-9}
2	3.4×10^{-10}	1.6×10^{2}	-3.4×10^{-10}
2.5	3.3×10^{-10}	2×10^{2}	-3.3×10^{-10}
3	2.9×10^{-10}	2.3×10^{2}	-2.9×10^{-10}
3.5	2.6×10^{-10}	2.7×10^{2}	-2.6×10^{-10}
4	3.6×10^{-10}	3.1×10^{2}	-3.6×10^{-10}
4.5	1.4×10^{-9}	3.8×10^{2}	-1.4×10^{-9}

δh in m (water table level above the river level)	**TELEMAC2D error in m^3 lost**	**ESTEL error in m^3 lost**	**Coupling error in m^3 lost**
5	1×10^{-14}	4.5×10^{2}	1×10^{-14}
5.5	8.2×10^{-10}	5.1×10^{2}	-8.2×10^{-10}
6	1.8×10^{-7}	5.6×10^{2}	-1.8×10^{-7}
6.5	1.8×10^{-9}	6.2×10^{2}	-1.8×10^{-9}
7.5	2.1×10^{-9}	6.4×10^{2}	-1×10^{-2}
8	2.2×10^{-7}	6.9×10^{2}	-2.2×10^{-7}
8.5	2.2×10^{-8}	7.2×10^{2}	1×10^{-14}
9	2×10^{-9}	8.2×10^{2}	1×10^{-14}
9.5	2.7×10^{-9}	8.8×10^{2}	-2.7×10^{-9}
10	3.7×10^{-9}	9.9×10^{2}	-3.7×10^{-9}

C. Results analysis

From Tab. 1, it appears that the major loss of mass is due to the computation done by ESTEL itself. It can be explained by the non-conservative discretization of the Richards equation we chose to use, the inherent non-linearity of this equation and the size of the mesh (about 3 or 4 meters). It seems that the loss of mass due to ESTEL is almost linear with respect to the difference between the water table and the well.

Concerning the loss of mass due to TELEMAC2D and the coupling, they appear to be almost equivalent and largely smaller (at least 10^{10} lower than ESTEL). The only exception can be found for δh = 7.5 m, where numerical instabilities appeared in the unsaturated part of the ground near the coupling surface, disturbing the accuracy of our coupling strategy. In fact, errors corresponding to TELEMAC2D and the coupling are always close to the machine error, contrary to ESTEL that gives an error about few percent of the initial volume of water.

IV. SIMPLIFIED RIVER CASE

A. Purpose and problem description

The aim of this case is to quantify the exchange between the river and the groundwater flow with respect to the discharge of the river. To do so, two hydrographs are used as input, one from the Var river (see Fig. 3) and one from the Rhine (see red curve in Fig. 10). As the bathymetry of our simplified river is different from those of the Var and the Rhine rivers, downstream conditions were adapted to fit the bathymetry.

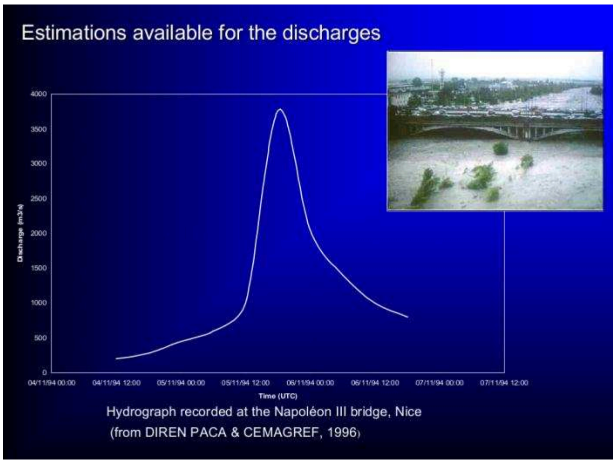

Figure 3. Hydrograph of the Var

The downstream condition is computed thanks to the Manning law:

$$Q = K_s * R_h^{2/3} * S * I^{1/2} \quad (6)$$

Where:

- K_s is the Strickler coefficient (-),
- R_h is the hydraulic radius (m),
- I is the slope of the hydraulic grade line (-),
- S is the section of the river (m^2).

Assuming that the soil is covered with cobbles with a grain diameter about 0.2 meter [8], the Ramette law [9] gives a Strickler coefficient of 33 for the riverbed. As the maximum value of the computed downstream condition is lower that the floodplain, there is no need to compute its value at this area.

The domain is 200 meters width, 50 meters length and 40 meters high (see Fig. 4). The riverbed is in the middle and has a depth of 20 meters and a width of 30. The flood plain has a depth of 10 meters and a width of 80. The slope of the riverbed and the floodplain is equal to 9×10^{-3} meter. Once again, the parameters of the Van Genuchten model are set to represent a loamy sand soil. The values of the parameters are identical to the previous test case.

The boundary conditions used in ESTEL is a Dirichlet conditions on the pressure head to model a fixed water table. More precisely, the water table is set at 18 meters high at the limit, whereas the river level evolves between 15 to 26 meters for the Var's hydrograph.

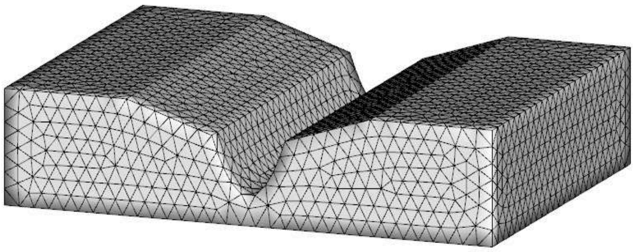

Figure 4. Mesh of the simplified river case

B. *Results*

It is important to mention that, as it can be seen in Figs. 5, 6, 8 and 9, there are two numerical instabilities at the entrance, where the water table is higher than the rest of the domain. It seems that few elements of the mesh used in ESTEL are over-constrained (no flux boundary conditions plus coupling term). However, neglecting these numerical errors, the water table level clearly follows the river level, showing that the results are relevant. It must be pointed out that the simulations presented in this section only correspond to the rising part of the hydrograph, as numerical instabilities previously mentioned lead to convergence issues at the discharge pick.

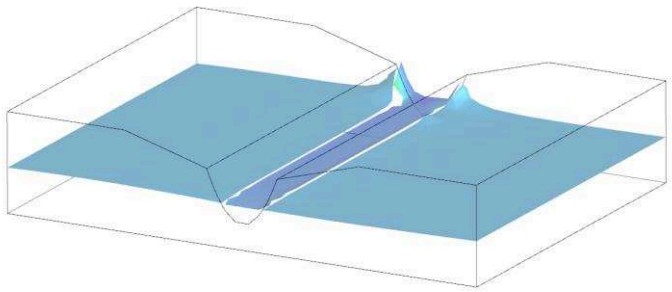

Figure 5. Level of the free surface of the river and the water table for the Var at t = 8.33 hours with a discharge of 408 $m^3.s^{-1}$.

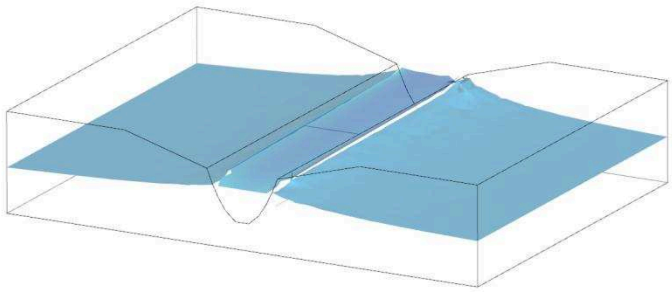

Figure 6. Level of the free surface of the river and the water table for the Var at t = 28.33 hours with a discharge of 2672 $m^3.s^{-1}$.

The results presented in Fig. 7 refer to the hydrograph of the Fig. 3. It appears that the discharge (curve in red) and the exchange between the river and the groundwater flow (curve in blue) have the same shape and are proportional. It is also important to mention that there is no delay in the exchange when the discharge abruptly increases around 23.6 hours. The amplitude of the exchange is rather large, with values from 1.1×10^{-2} to 1.8×10^{-1} $m^3.s^{-1}$.

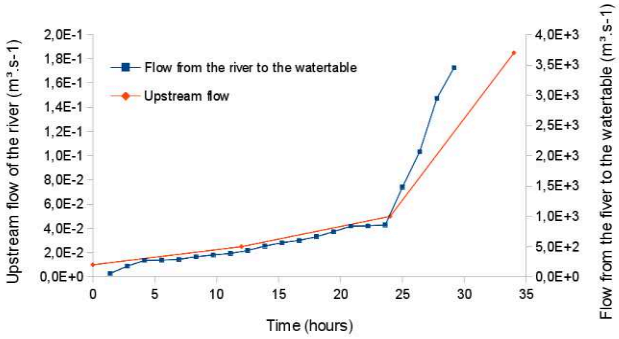

Figure 7. Comparison of the discharge and the exchanges between the water table and the river with the Var's hydrograph

The results presented in Fig. 10 correspond to the Rhine river. Similarly to the results of the Var, the discharge (curve in red) and the exchange between the river and the groundwater flow (curve in blue) have the same shape and are proportional. The amplitude of the exchange is smaller compared to the Var, with values from 4×10^{-2} to 6×10^{-2} $m^3.s^{-1}$.

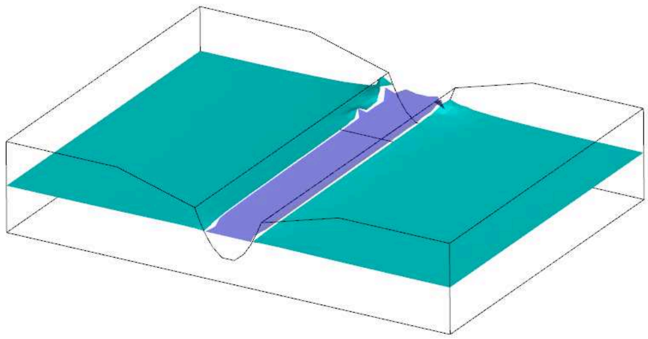

Figure 8. Level of the free surface of the river and the water table for the Rhine at t = 2 hours with a discharge of 1205 $m^3.s^{-1}$.

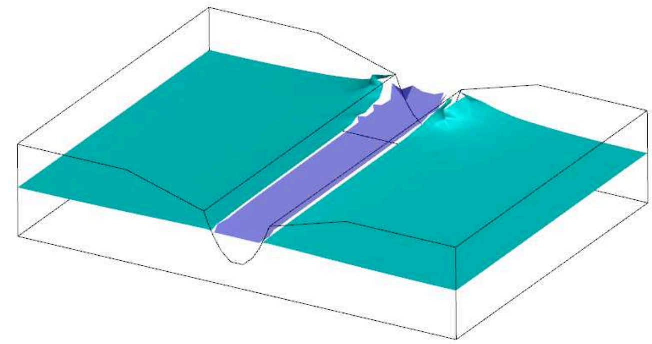

Figure 9. Level of the free surface of the river and the water table for the Rhine at t = 14.4 hours with a discharge of 1610 $m^3.s^{-1}$.

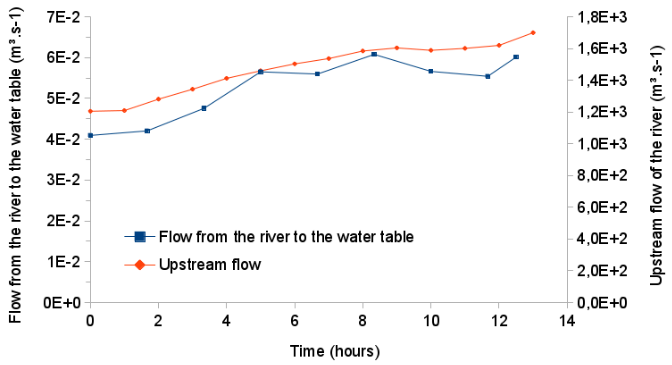

Figure 10. Comparison of the discharge and the exchanges between the water table and the river with the Rhine hydrograph.

C. Results analysis

The results of these simplified river simulations clearly show that the exchange between the ground water flow and the river depend on the discharge of the river. As the amplitude of the exchange could be large (with a factor from 1 to 16 with the Var), it suggests that a constant flux to model the exchange would not have been relevant.

From our experience, the numerical stability of this type of simulation is a real issue. We observed that it is mandatory to compute a stable and continuous initial condition. To do so, we first compute a stationary solution of the river flow without the coupling. Then, we create a continuous pressure field in ESTEL, based on the boundary conditions on the water table and the height of the river.

V. Discussion

In this paper we have presented our iterative coupling strategy based on the use of TELEMAC2D and ESTEL. Their respective governing equations, the considered assumptions as well as the details of the iterative coupling procedure were presented. A pseudo Artesian well was simulated to validate the coupling procedure and quantify the error in terms of mass loss. Results showed that the major loss of mass is due to the computation done by ESTEL (few percents of the mass) while errors related to TELEMAC2D and the coupling are negligible. Finally, the exchange between a simplified river and a groundwater flow was quantified with respect to the discharge of the river. The results showed that the exchange could not be model by a constant flux. It appeared that the exchange depends on the discharge and that its amplitude could be large. It clearly shows that this type of coupling strategy can be used to improve the accuracy of simulated surface flows, as the interaction between the river and the groundwater flow are modeled.

In order to simulate more realistic cases, several developments are still required. Regarding the physical aspects, it would be important to model runoff phenomenon in order to improve the accuracy of the simulated exchange. It could also be interesting to study the impact of a clogging layer [10] that appears at the interface between the river and the groundwater table. As the numerical stability is still a real issue, it would be important to investigate the loss of mass due to ESTEL and to vanish instabilities that could appear near the entrance of the river. The computational cost could also be prohibitive for more realistic cases, with actual computation time about a day or a week, parallel computation would be relevant. Finally, our actual procedure to create the geometry is not generic enough.

A new tool allowing the automatic generation of the ESTEL (3D) mesh from the TELEMAC2D (2D) is currently under development. This tool generates automatically the 3D mesh (tetrahedron) and the interface between the 2D and 3D meshes as well (see Fig. 11). This tool overcomes one of the biggest pre-processing tasks and procures exact nodal interpolation between these two meshes, and hence improves significantly the coupling algorithm. Moreover it would be a great benefit giving the user the possibility to model interactions at any place of the rivers without creating manually the mesh, indeed most of rivers are already in the TELEMAC2D databank.

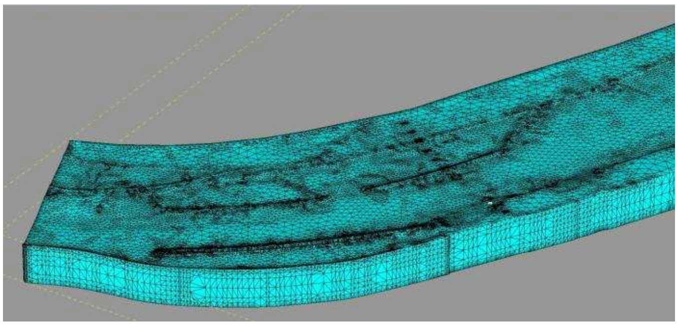

Figure 11. Example of extrapolation of a 2D mesh used in Telamac2D.

Acknowledgements

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